Qubits, Cats and Gates

Oxford Ion Trap QC project: status and plans

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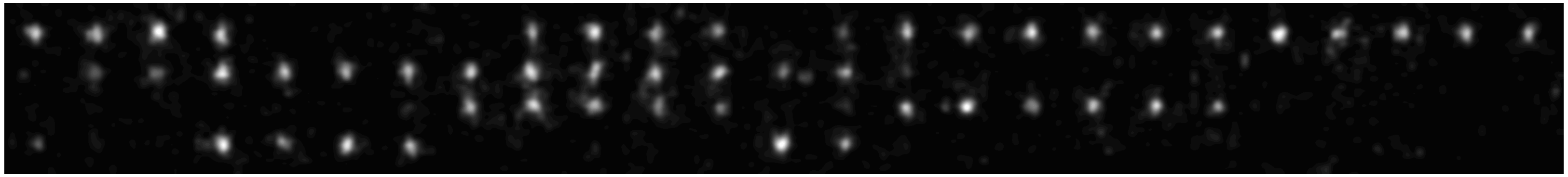
www.physics.ox.ac.uk/users/iontrap

$ DTO / ARO / NSA
£ Royal Society, EPSRC
€ ConQUEST, SCALA networks



Recent results (year to July 2005)

Ion cooling:

- single ion cooled to ground motional state: $\langle n \rangle = 0.02(2)$, $T = 10\mu K$
- very low motional heating rate: 3(1) phonons/sec
- long motional decoherence time: ~0.2 sec
- strings of 2 and 3 ions cooled close to motional ground states

Single-qubit coherent manipulations:

- Rabi flops, Ramsey & spin-echo sequences with up to 98% fidelity
- "Schrödinger Cat" spin-motion entangled states with $\langle n \rangle = \alpha^2 = 9$
- new, long-lived, qubit in $^{43}Ca^+$, decoherence time $T_2 = 0.9(2)$ sec

Two-qubit logic gate:

- deterministic entanglement (& tomography) of 2 ions: 82(2)% fidelity

Oxford ion trap group

B.K. D.L. J.H.

Faculty:
Andrew Steane
Derek Stacey
David Lucas

Post-docs:
Jonathan Home
Matt McDonnell
Nick Thomas
+ ???

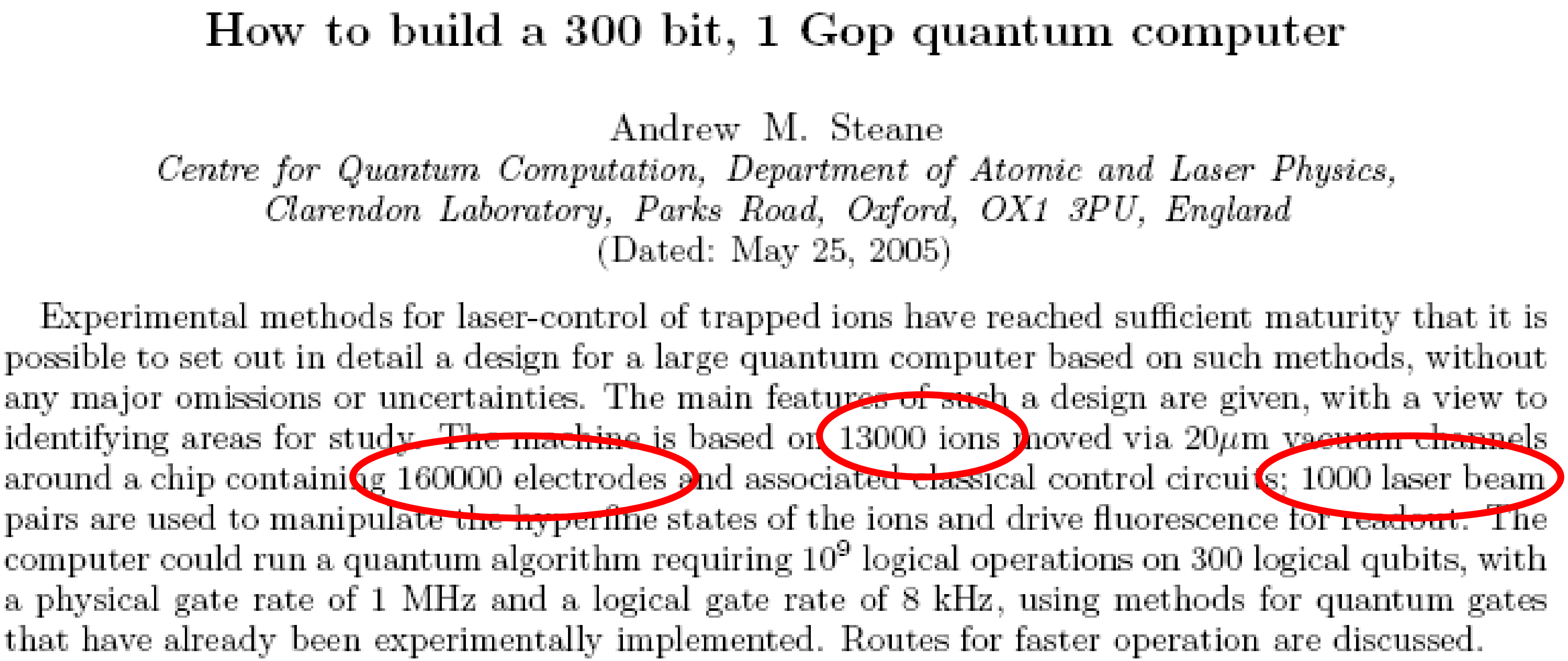
Ph.D. students:
Ben Keitch
Greg Imreh
David Szwer

Posters M7 and M8

Overview

- Background
  - choice of ion (and a recent surprise!)
- Qubits
  - $^{40}Ca^{+}$ ground-level spin qubit ($S_{1/2}$ m_S=±1/2) and readout
  - $^{43}Ca^{+}$ hyperfine qubit ($S_{1/2}$ F=3, F=4)
  - preparation, readout and detection issues for $^{43}Ca^{+}$
- Cats
  - "Schrödinger Cat" spin-motion entangled states
  - pushing entangled states beyond the Lamb-Dicke regime
- Quantum logic gates
  - deterministic two-ion entanglement with tomography
  - preliminary studies for fast gates using a pulsed force
- [Multiple traps & scaling up]
  - design studies [Home & Steane quant-ph/0411102]
  - multiple trap under construction

The vision…


How to build a 300 bit, 1 Gop quantum computer

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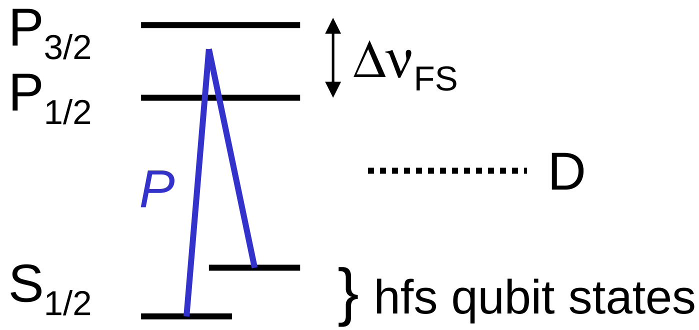
(Dated: May 25, 2005)

Experimental methods for laser-control of trapped ions have reached sufficient maturity that it is possible to set out in detail a design for a large quantum computer based on such methods, without any major omissions or uncertainties. The main features of such a design are given, with a view to identifying areas for study. The machine is based on 13000 ions moved via 20μm vacuum channels around a chip containing 160000 electrodes and associated classical control circuits; 1000 laser beam pairs are used to manipulate the hyperfine states of the ions and drive fluorescence for readout. The computer could run a quantum algorithm requiring 10^9 logical operations on 300 logical qubits, with a physical gate rate of 1 MHz and a logical gate rate of 8 kHz, using methods for quantum gates that have already been experimentally implemented. Routes for faster operation are discussed.

F ~ 0.9999 (ish)

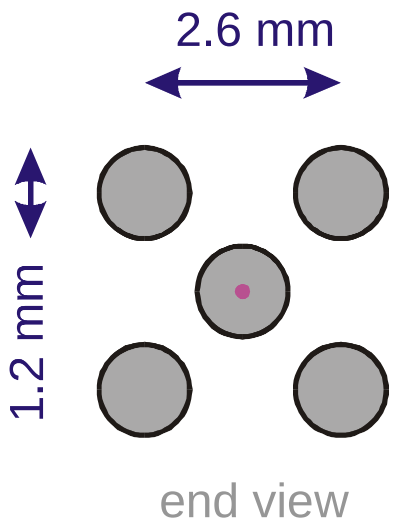
Choice of ion

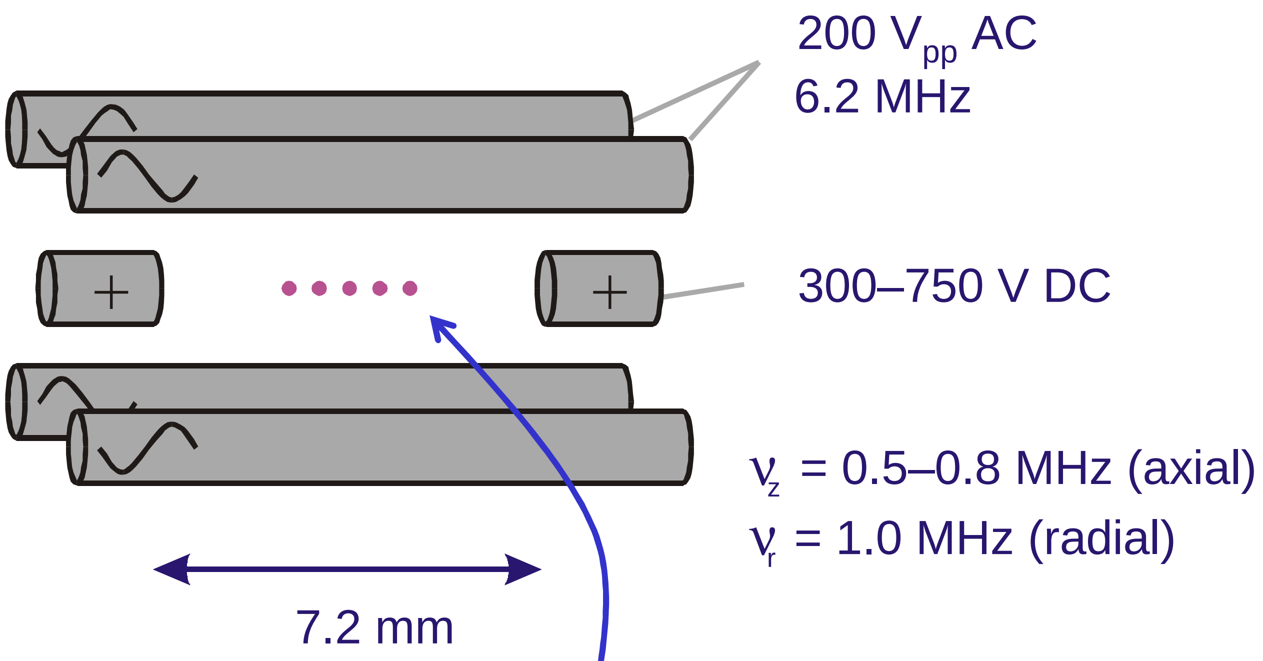


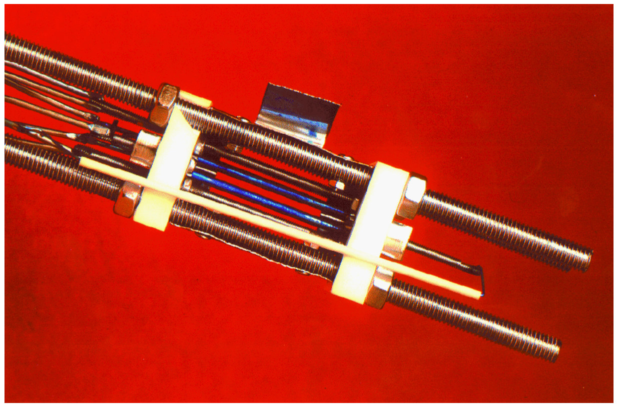
ion		$^9Be^+$	$^{43}Ca^+$	$^{111}Cd^+$
λ ($S_{1/2}$-$P_{3/2}$)	nm	313	393	215
λ' ($D_{3/2}$-$P_{1/2}$)	nm	-	866	-
fine structure $\Delta\nu_{FS}$	THz	0.20	6.7	75
linewidth Γ(S-P)	MHz	20	23	47
~~“quality factor” $\Delta\nu_{FS}/4\Gamma$~~	~~10^4~~	~~0.25~~	~~7.3~~	~~40~~
$(1/\varepsilon) \propto \lambda^3 P / \Omega_R$	10^4	25	49	8.1

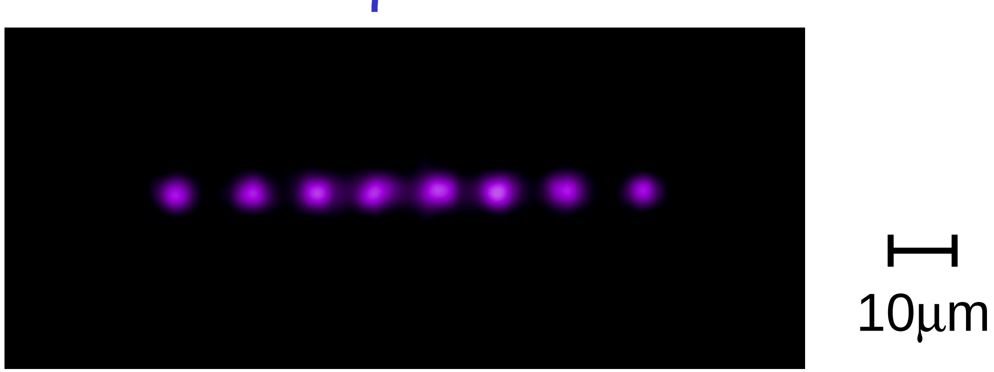
Ozeri *et al.*, PRL 2005

Linear ion trap

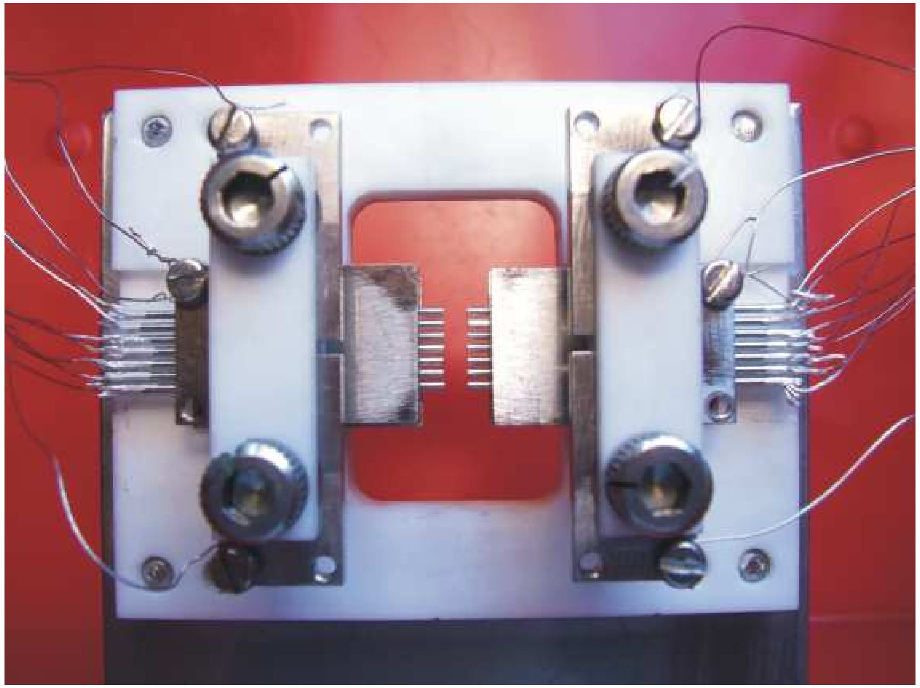




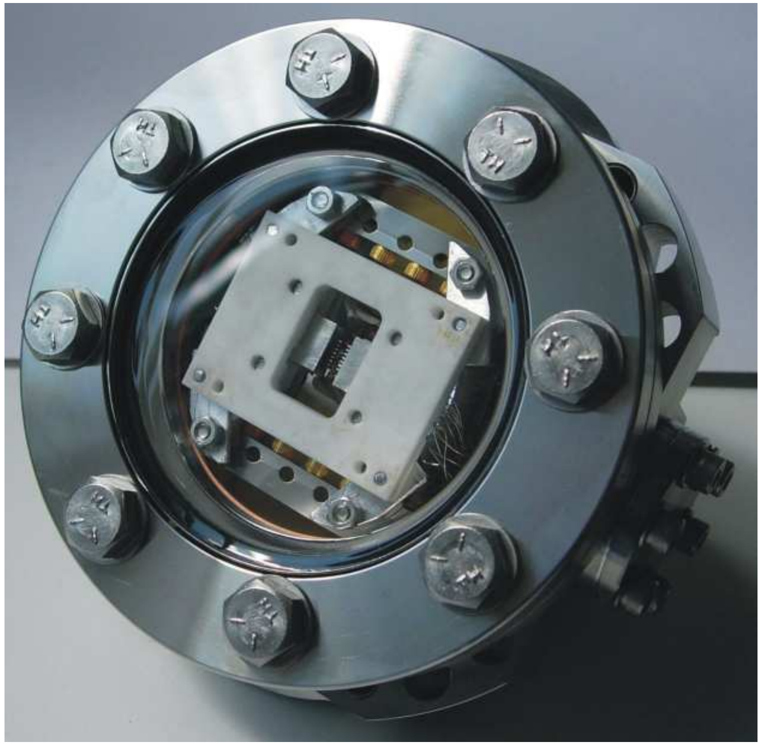




Under (macro)fabrication!

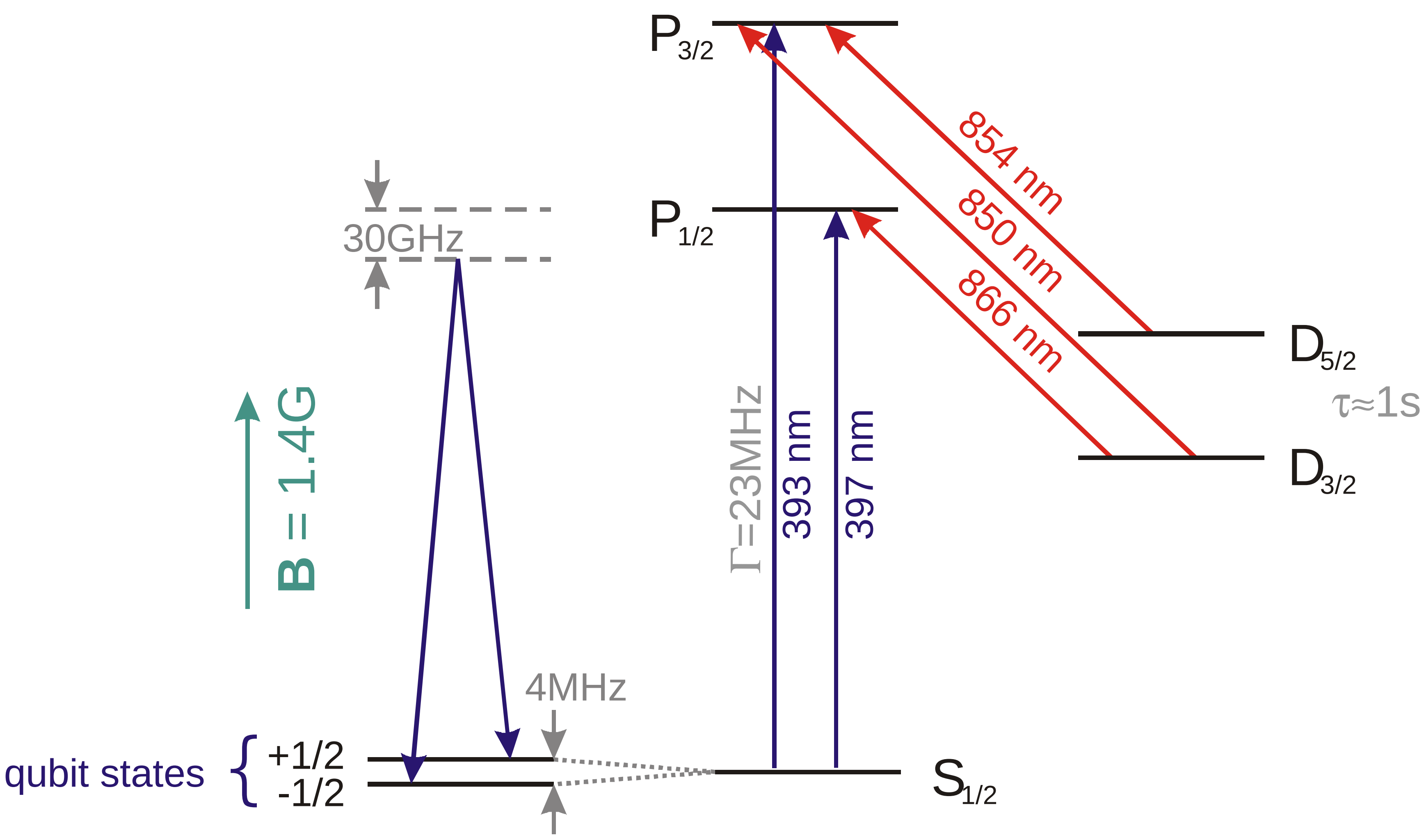


ion-electrode distance = 0.7 mm
trap-trap separation = 0.8 mm
test open design concept
Built by University of Liverpool (S.Taylor)

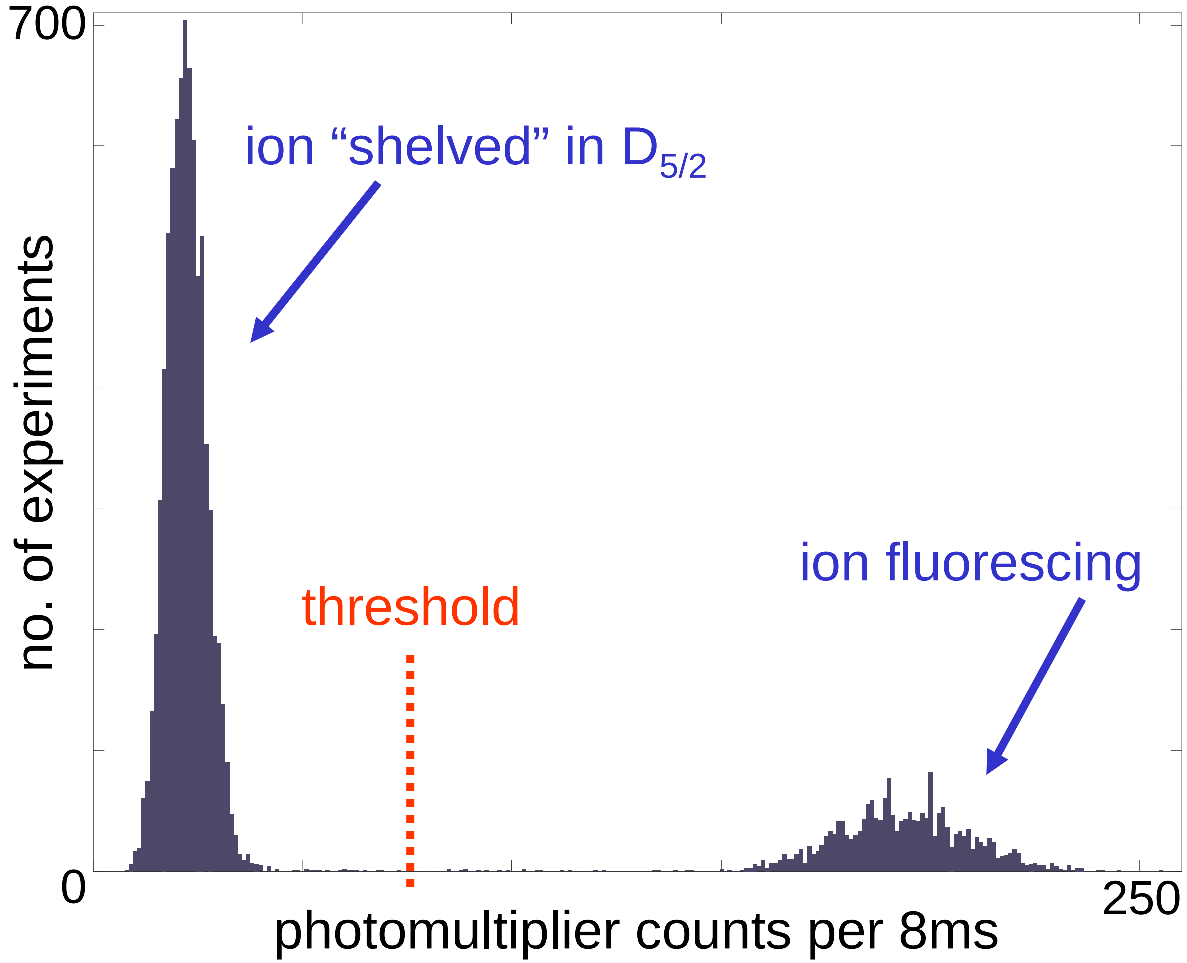


Qubits

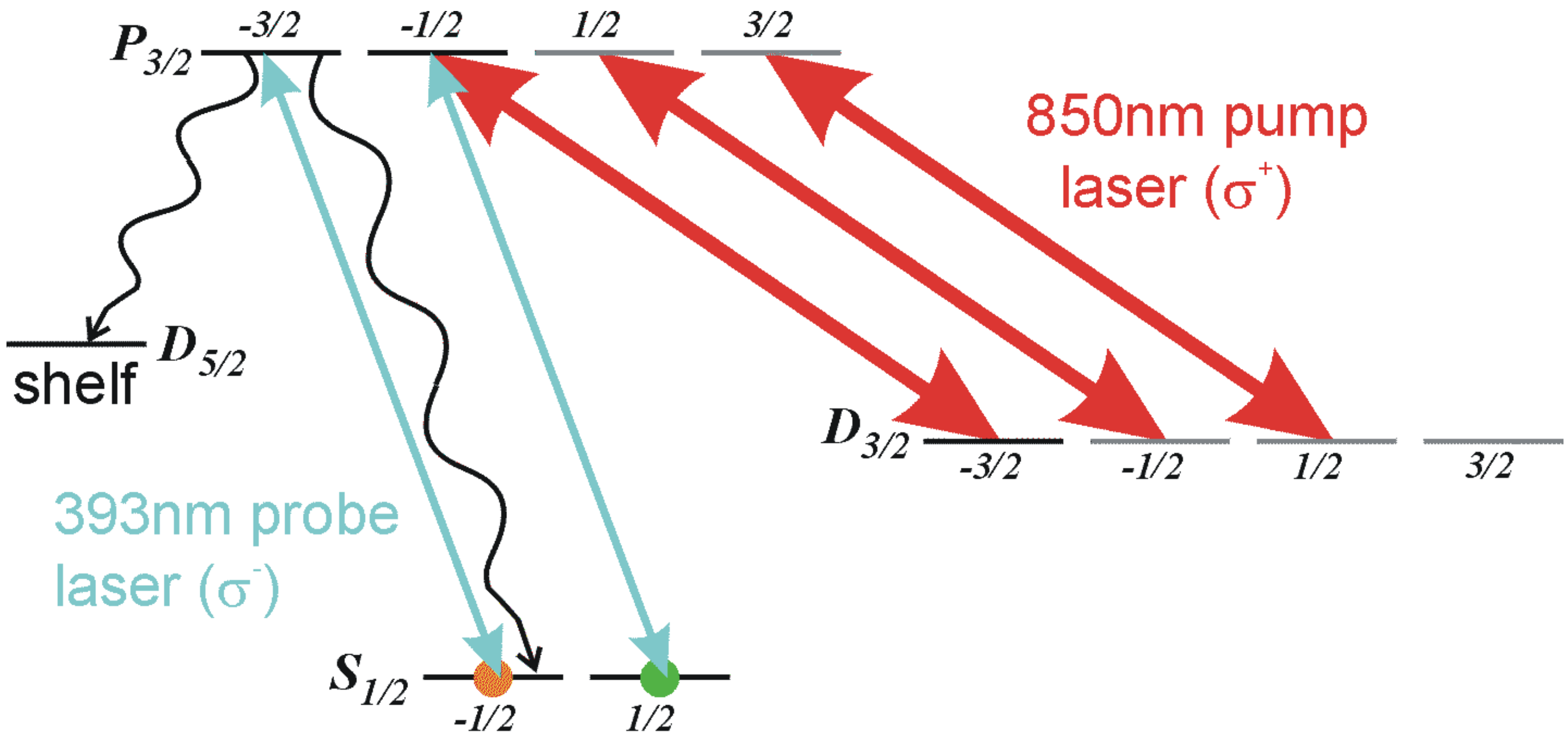
$^{40}Ca^{+}$ level diagram



Efficient state detection

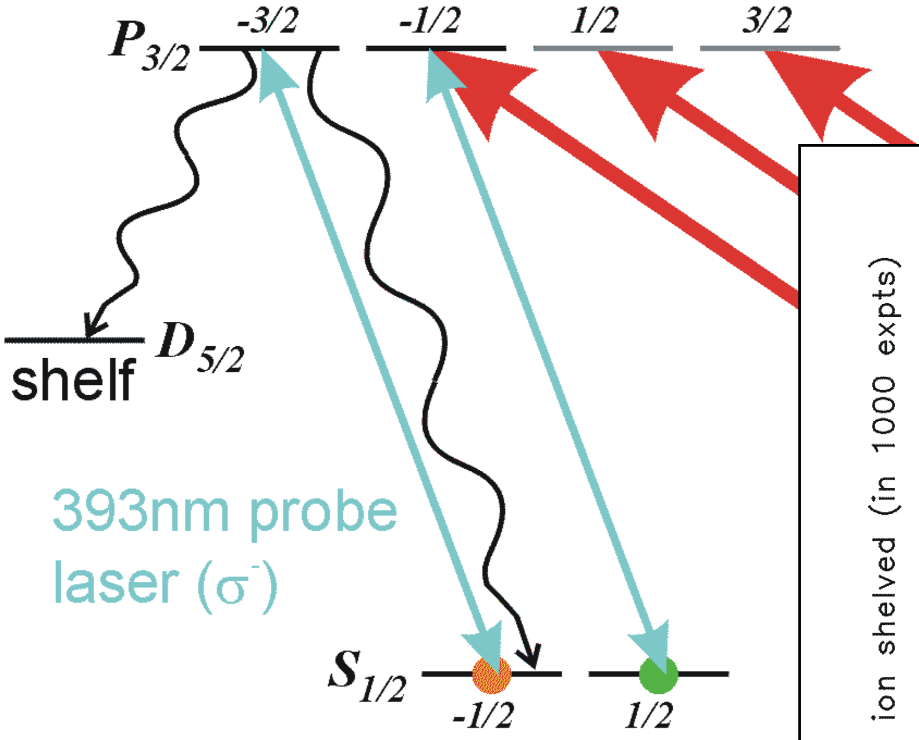


"E.I.T." read-out

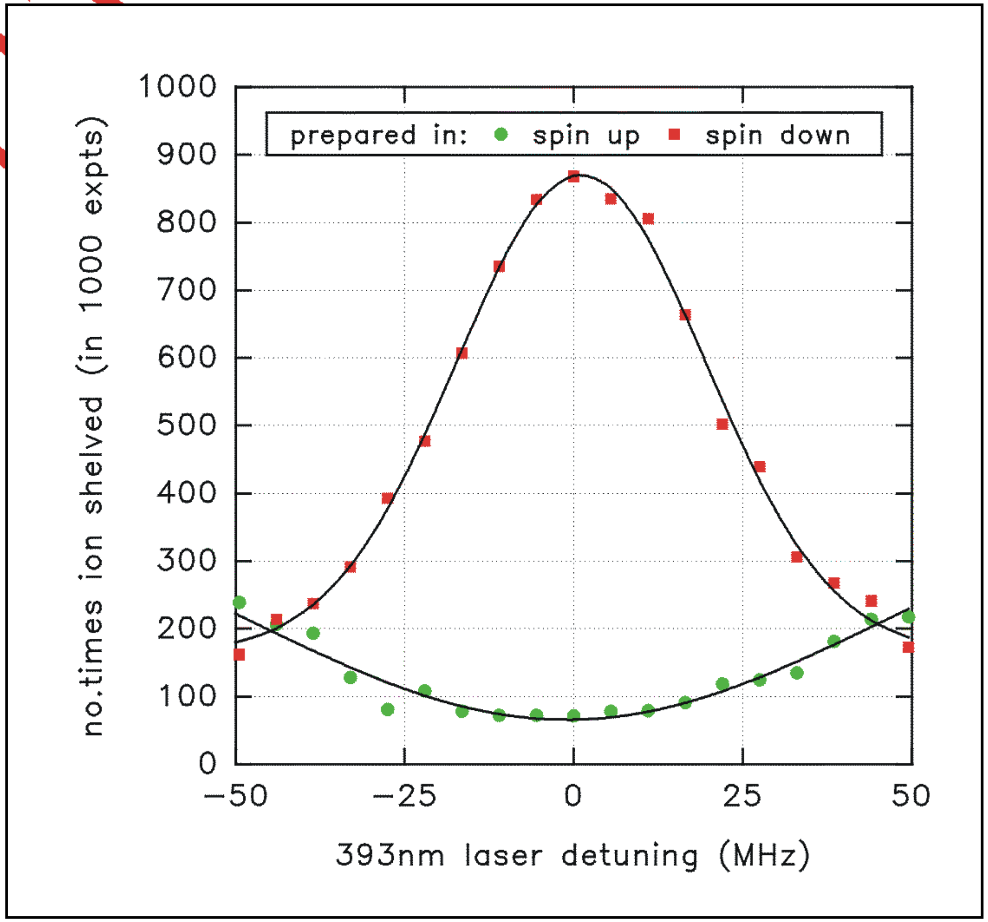


$P_{3/2}$
-3/2
-1/2
1/2
3/2
850nm pump
laser (σ^+)
$D_{5/2}$
shelf
$D_{3/2}$
-3/2
-1/2
1/2
3/2
393nm probe
laser (σ^-)
$S_{1/2}$
-1/2
1/2

"E.I.T." read-out

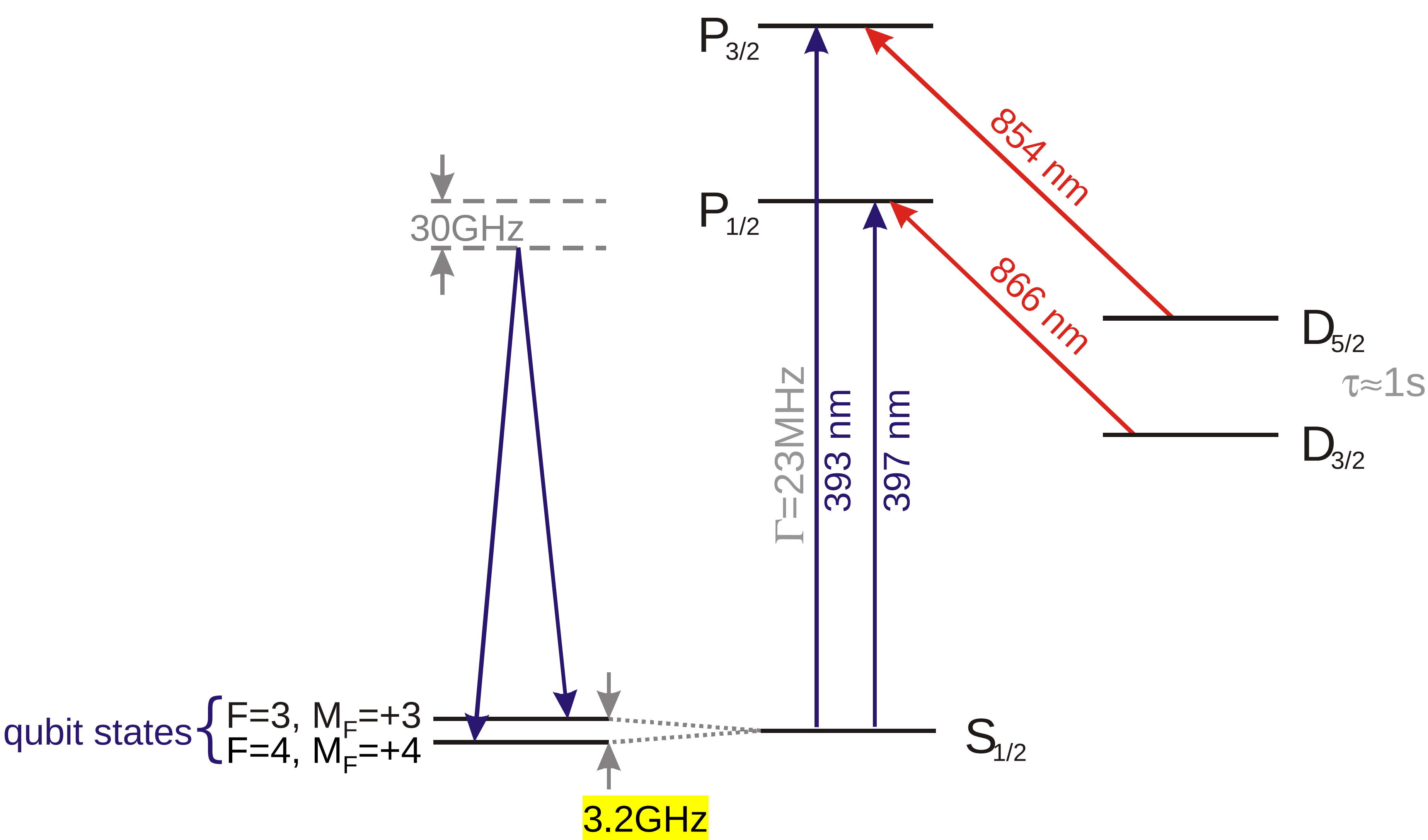


- 90% qubit readout efficiency
- measurement of spin direction without magnetic moment!

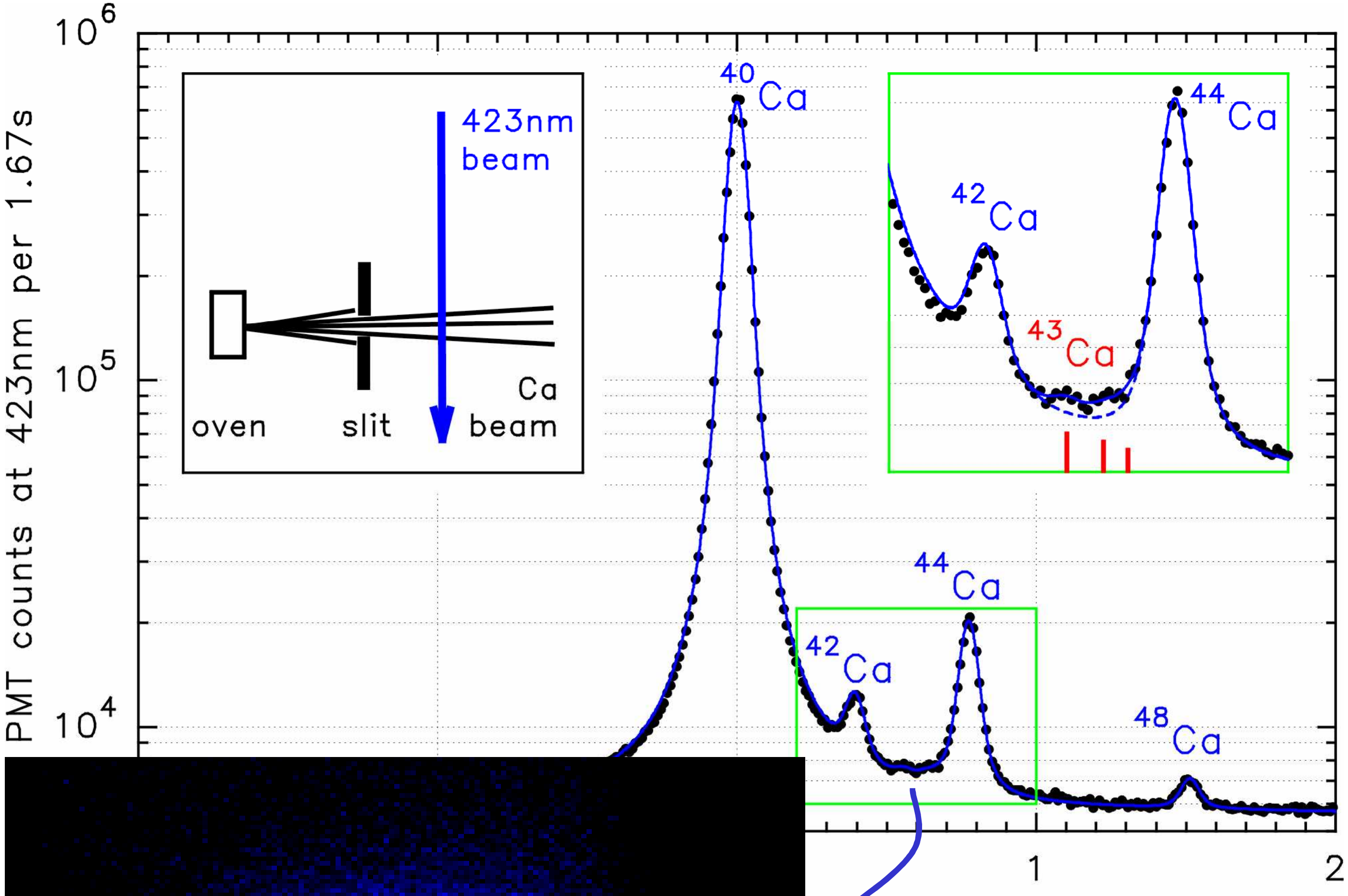


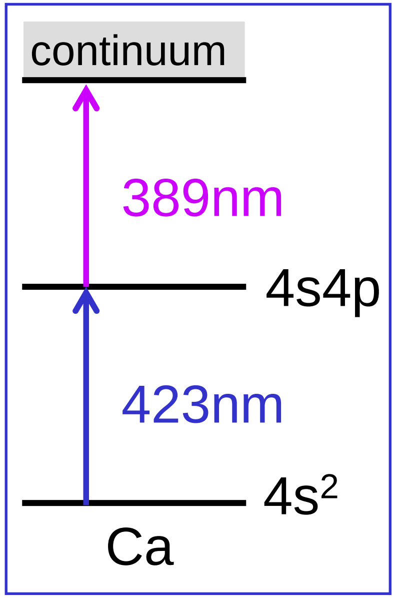
McDonnell *et al.* PRL 2004

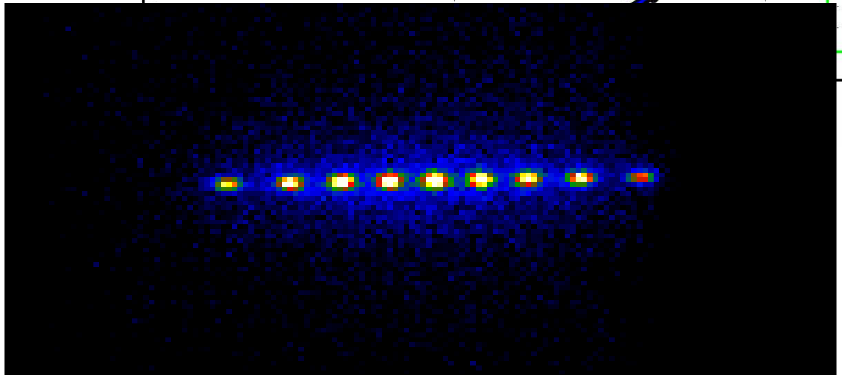
$^{43}Ca^+$ level diagram



Isotope-selective photo-ionization



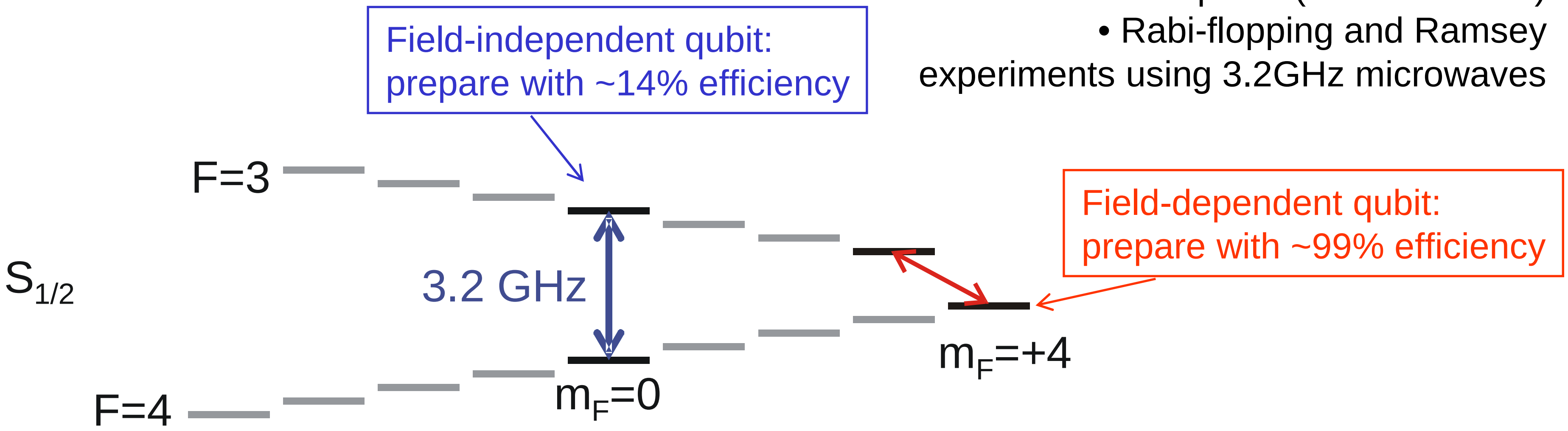


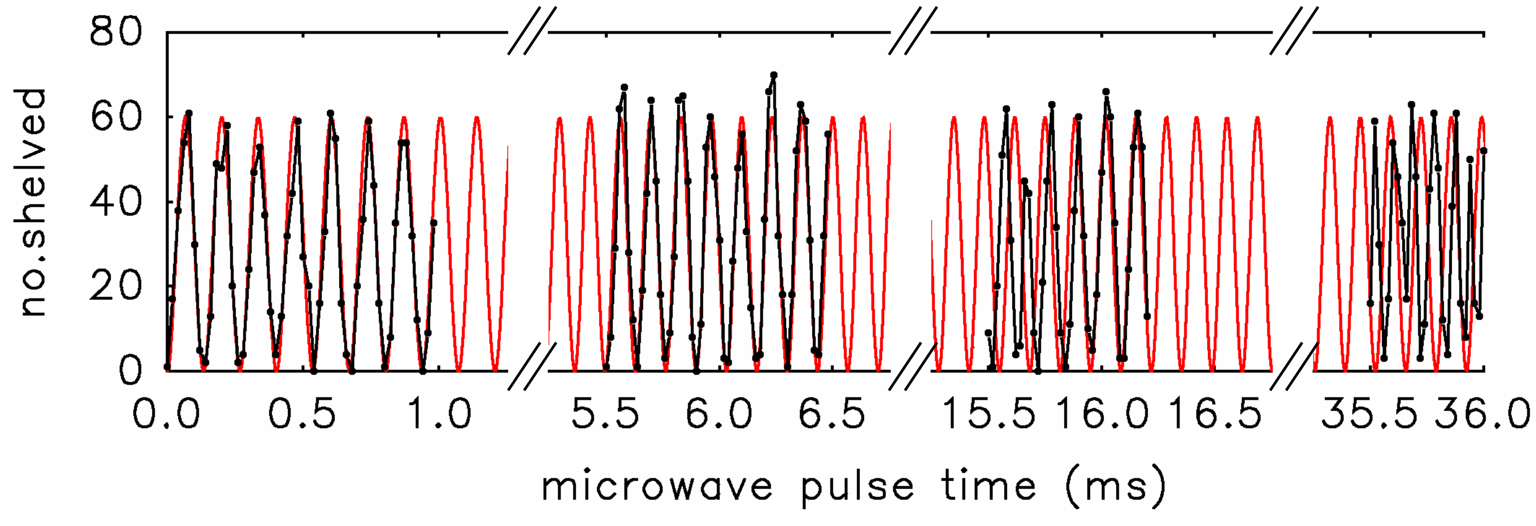


Kjaergaard *et al.*, Appl.Phys.B 2000
Gulde *et al.* Appl.Phys.B 2001
Lucas *et al.*, PRA 2004

Hyperfine qubits in $^{43}Ca^{+}$

- readout by shelving with single 393σ+ pulse (~95% efficient)
- Rabi-flopping and Ramsey experiments using 3.2GHz microwaves

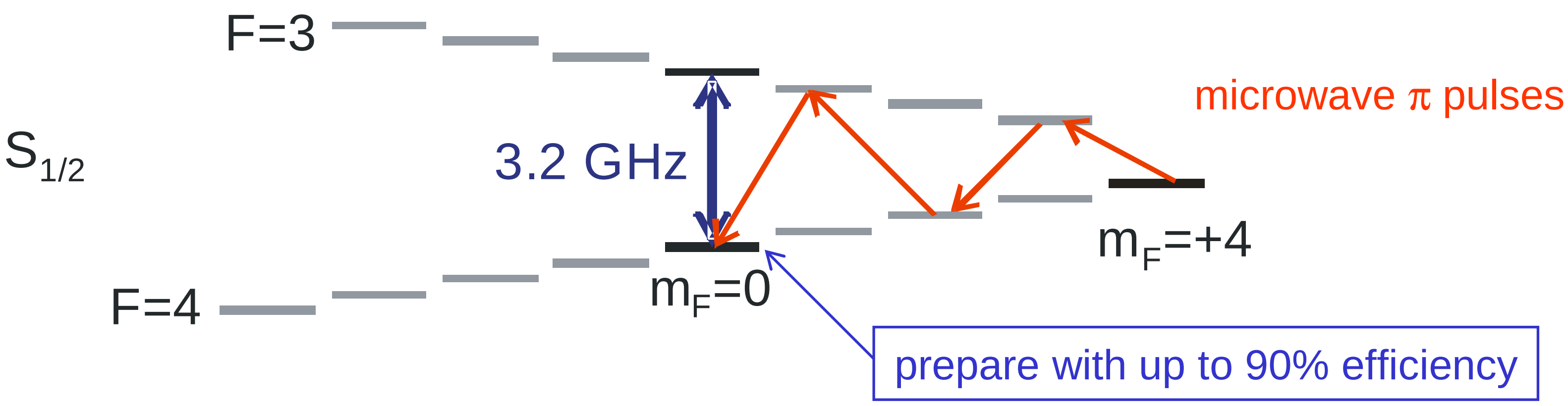


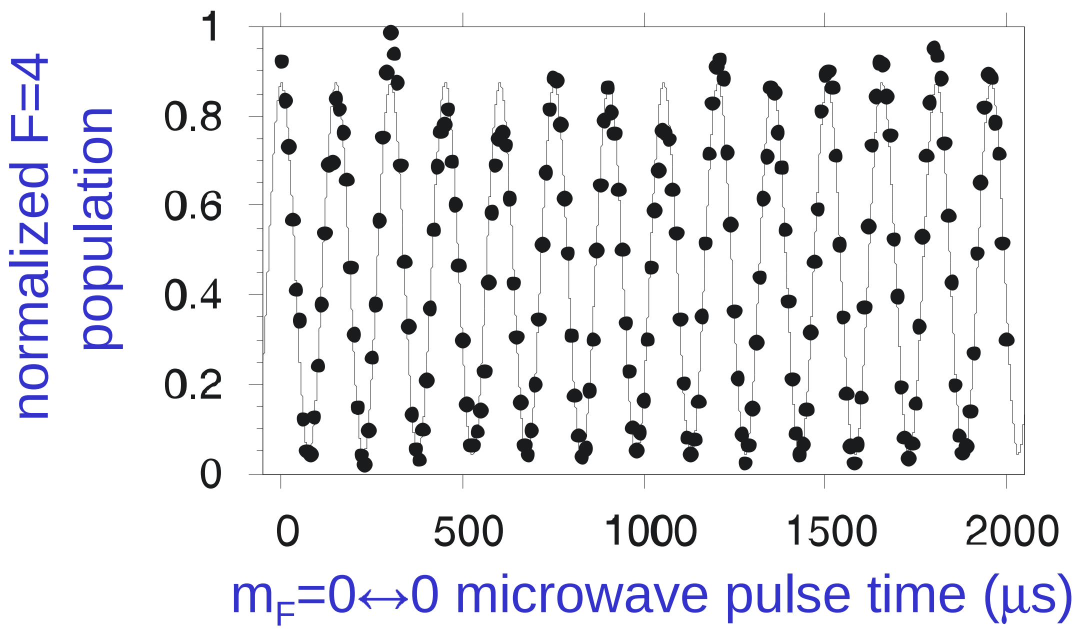


Rabi flops on 0–0 qubit

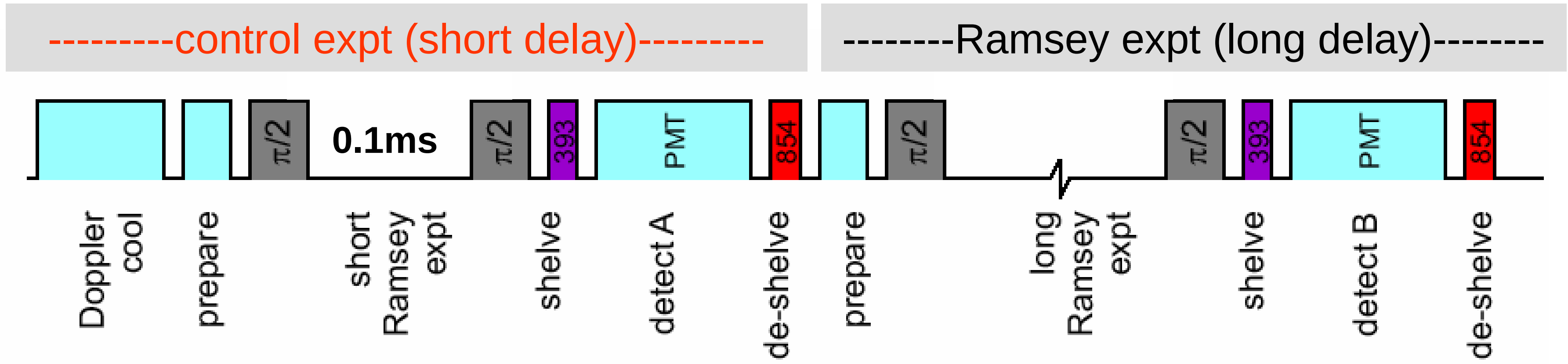
(~270 flops)

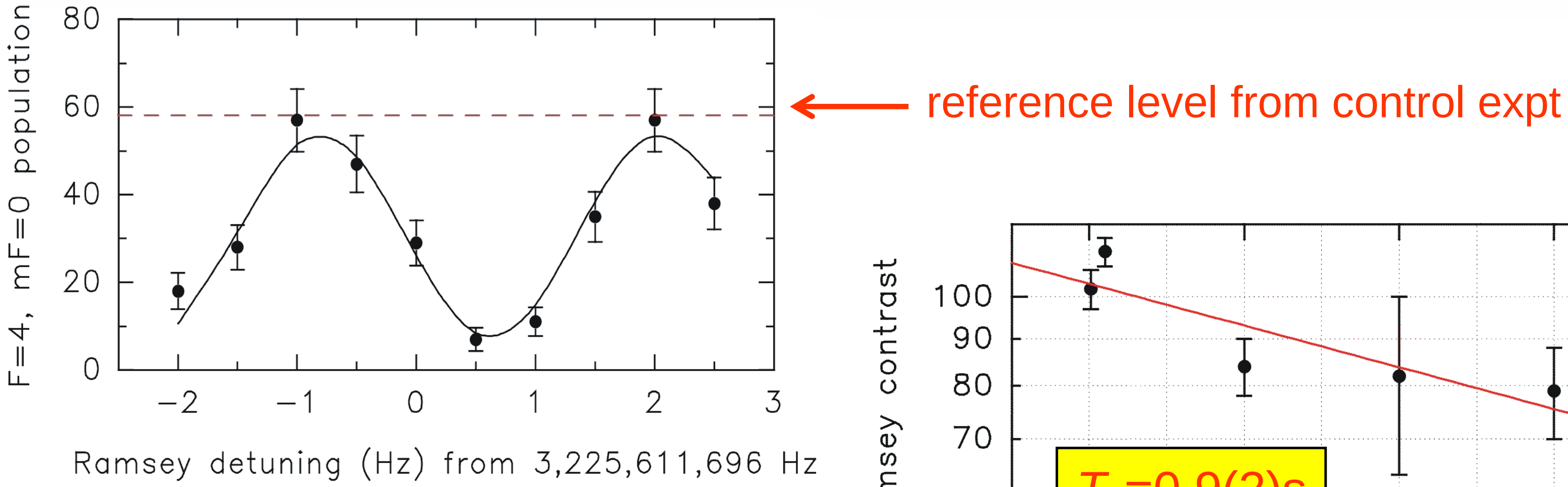
Hyperfine qubits in $^{43}Ca^{+}$

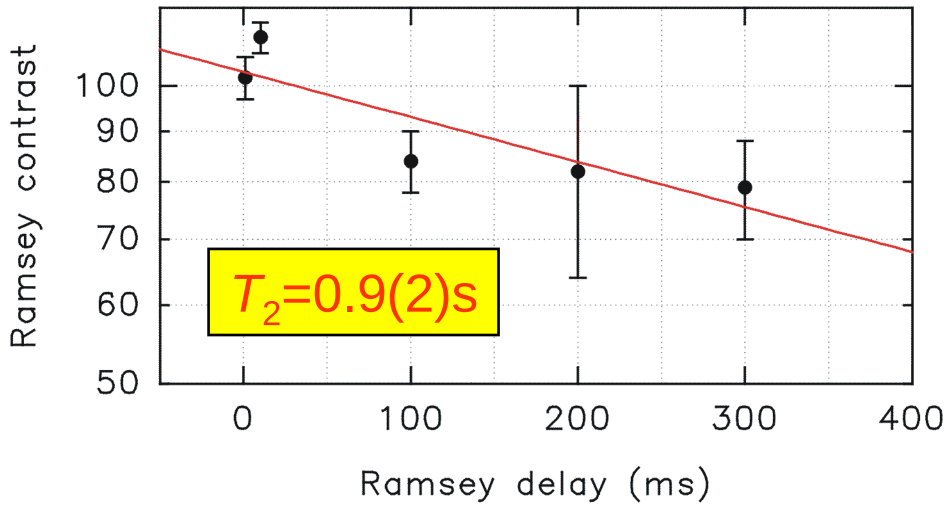




Long-lived qubit in $^{43}Ca^+$

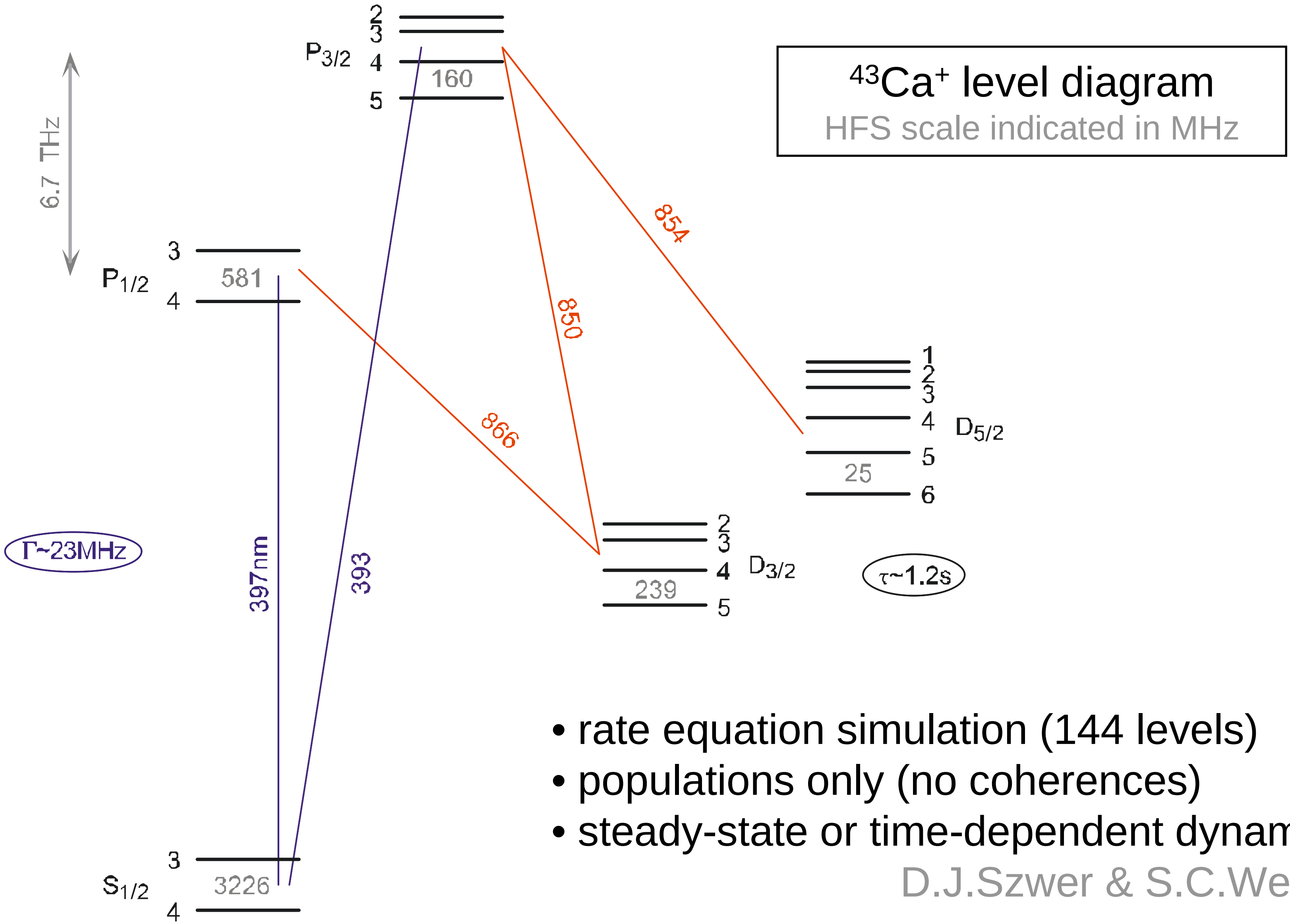






At working field of 1.7G, residual field-sensitivity is ~4Hz/mG.

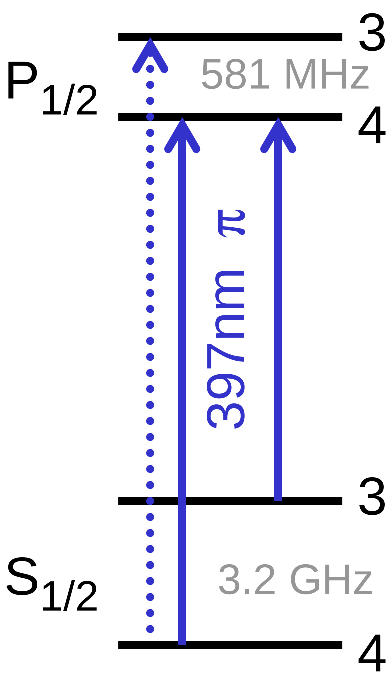
Preparation, readout, detection in $^{43}Ca^+$



- rate equation simulation (144 levels)
- populations only (no coherences)
- steady-state or time-dependent dynamics

D.J.Szwer & S.C.Webster

"Clock" qubit initialization in $^{43}Ca^{+}$



Optical pumping:

$m_F=0 \not\rightarrow m_F=0$ if $\Delta F=0$
so $S_{1/2}(4,0)$ should be a dark state…

BUT off-resonant F=4 → F=3 pumps ion out of (4,0) clock state.

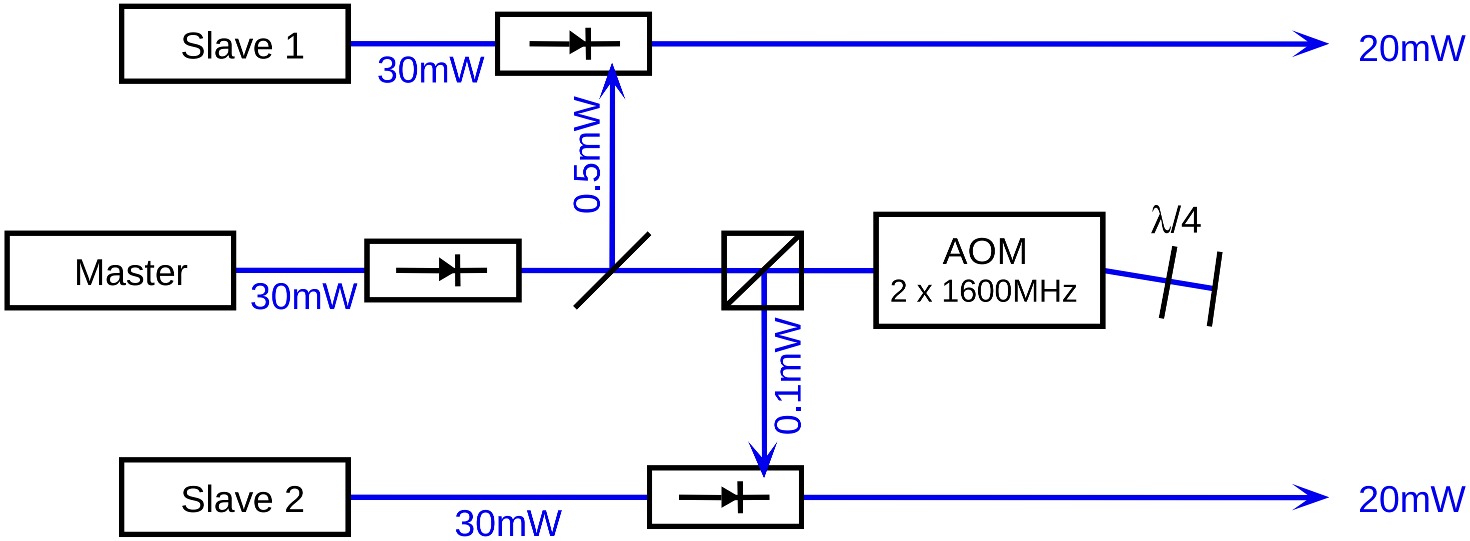
Best available optical pumping efficiencies:

98.9% (I_{397}=0.01, takes 3.2ms!)

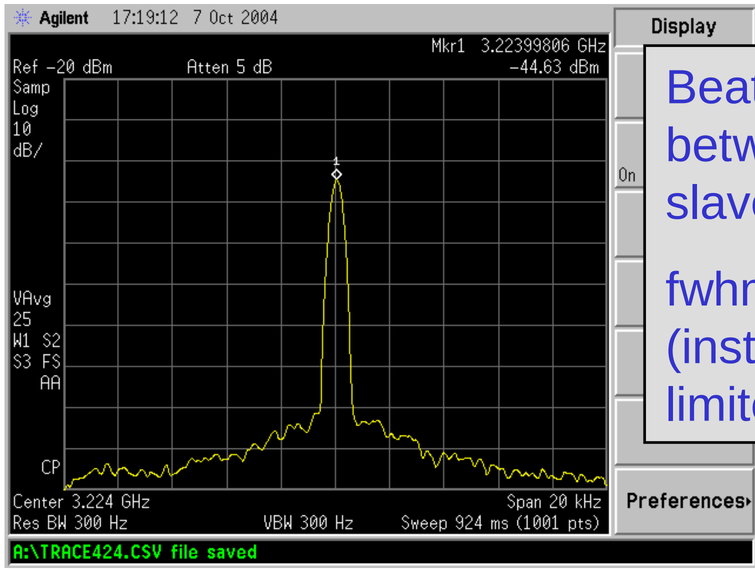
98.6% (I_{397}=0.1, takes 100μs)

[Note preparation of (4,+4) stretched state is only limited by σ+ polarization purity.]

Raman laser system for $^{43}Ca^{+}$



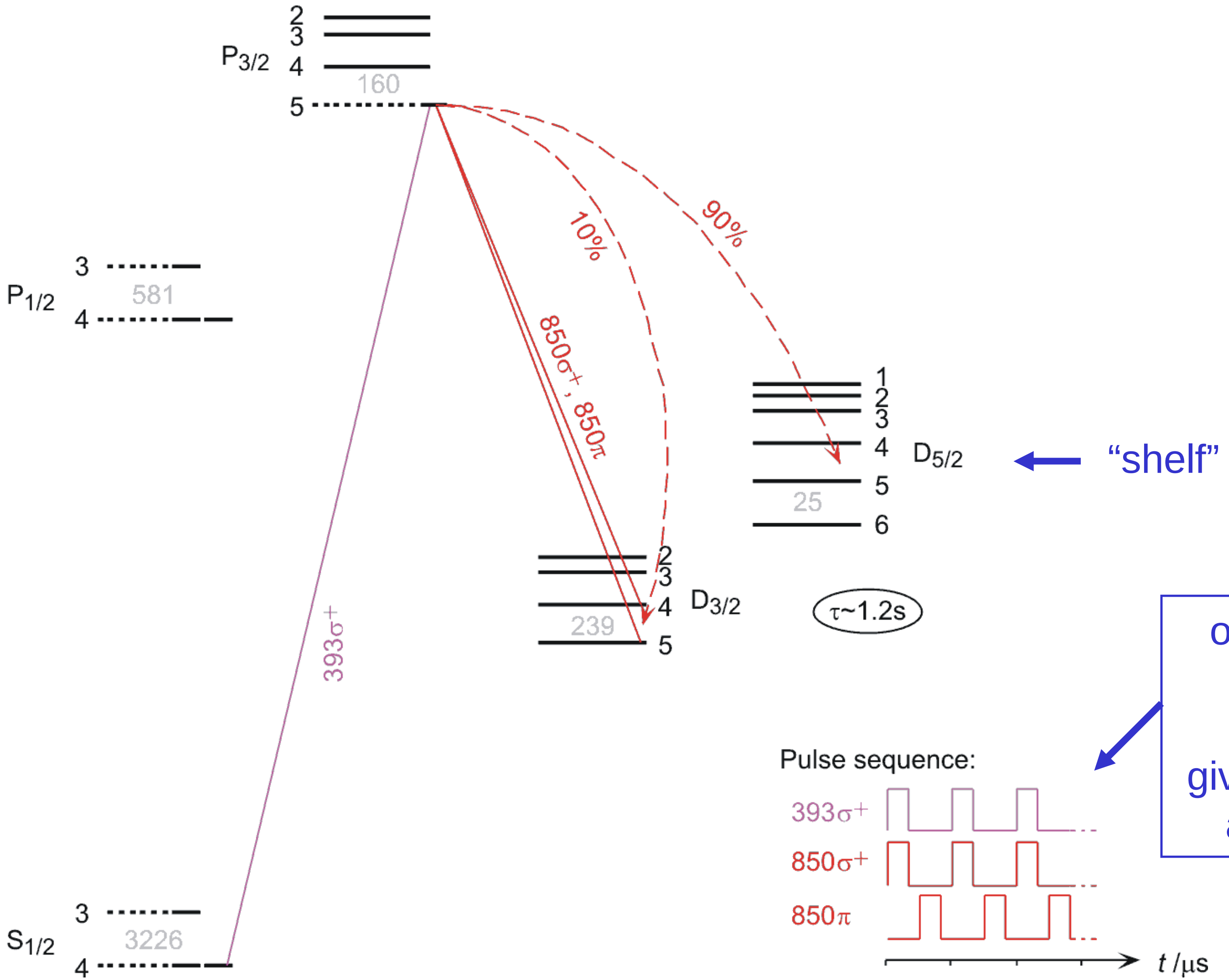
- need phase-locked 3.2GHz offset for hyperfine structure qubit
- 20mW usable slave output from 0.05mW injection beam
- AOM efficiency ~25% per pass



Beat signal between slave beams

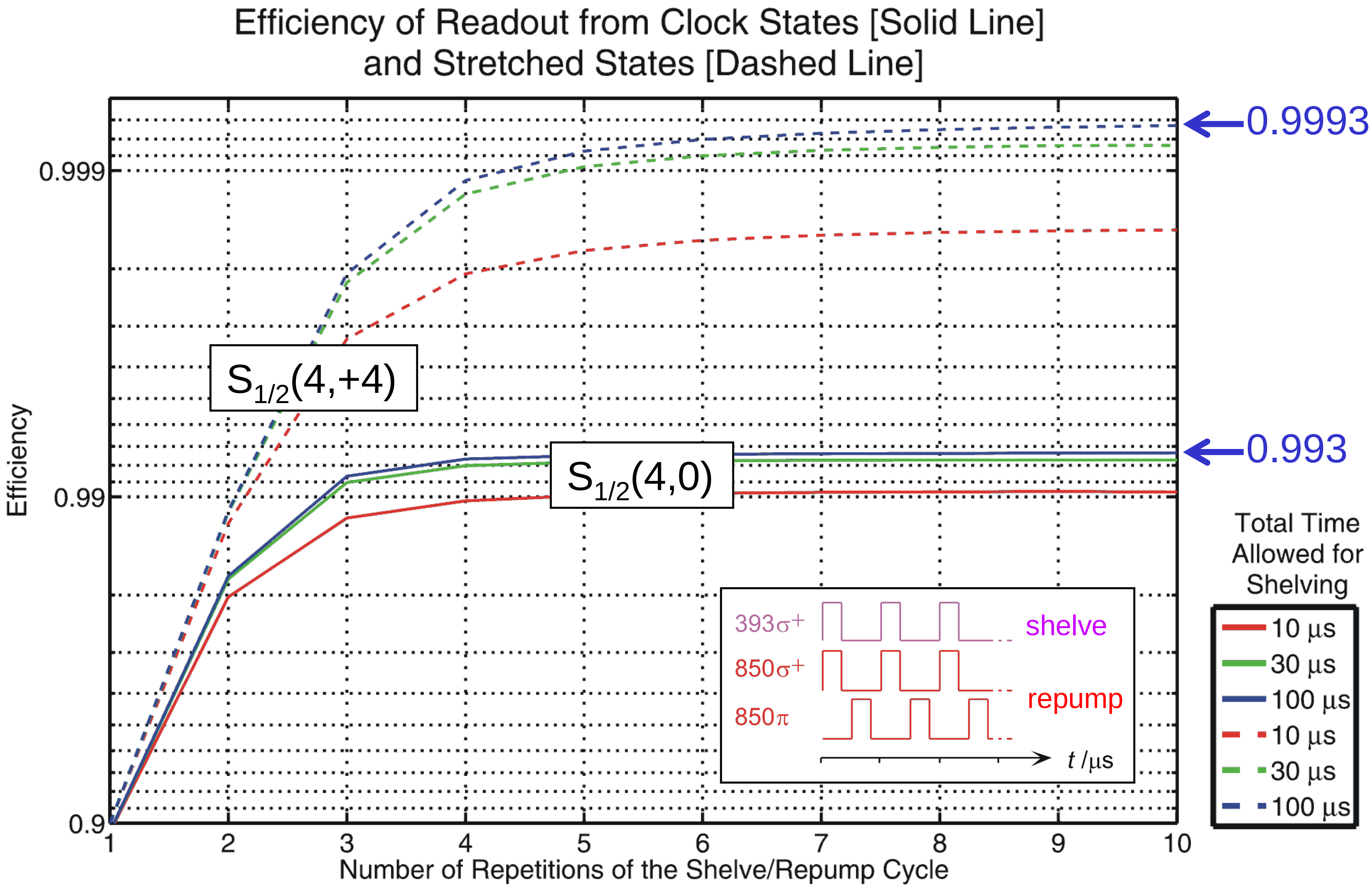
fwhm=10Hz (instrument limited)

Shelving for qubit readout in $^{43}Ca^{+}$

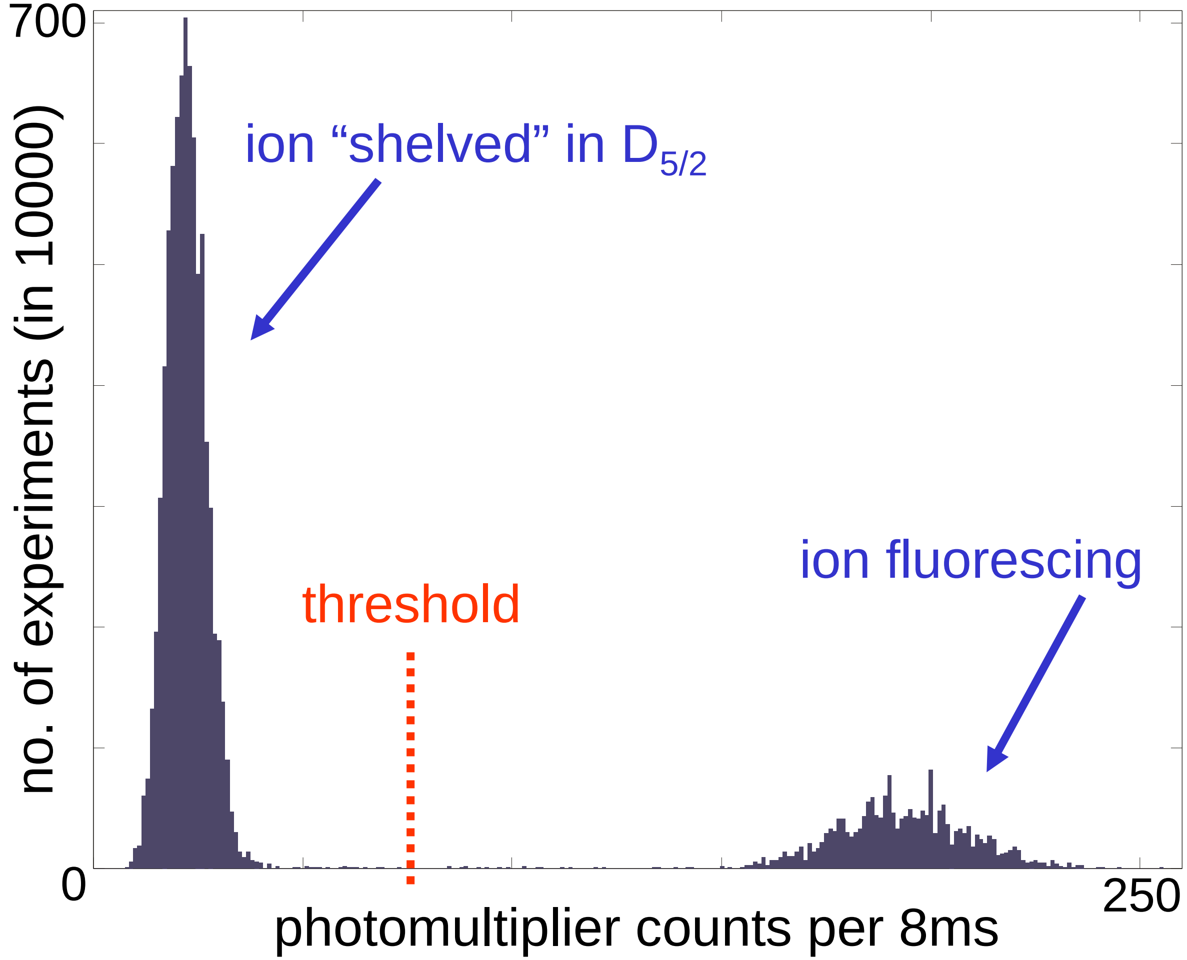


optimize pulse lengths and intensities for given no.pulses and total time

Shelving for qubit readout in $^{43}Ca^{+}$

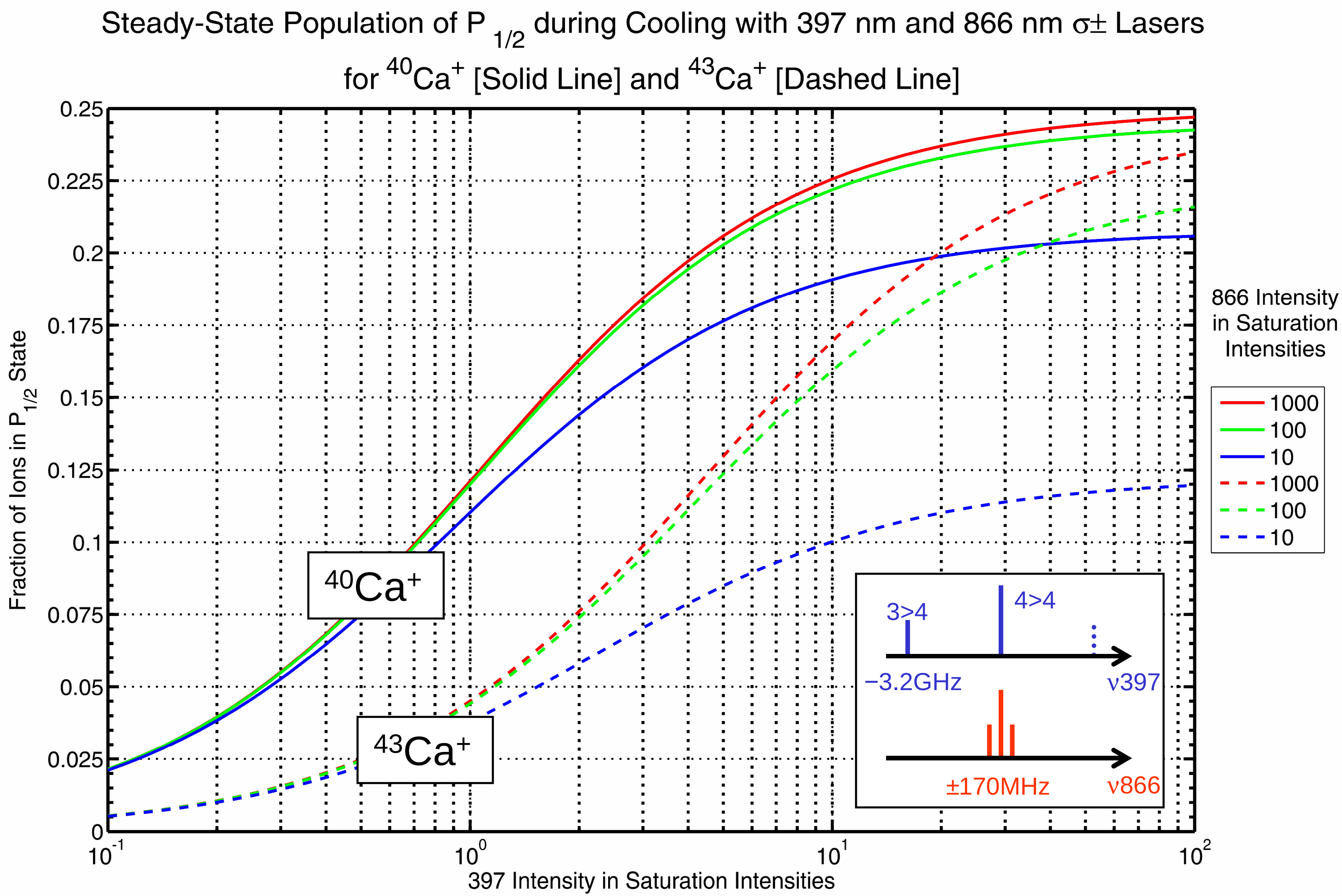


Efficient state detection in $^{40}Ca^{+}$

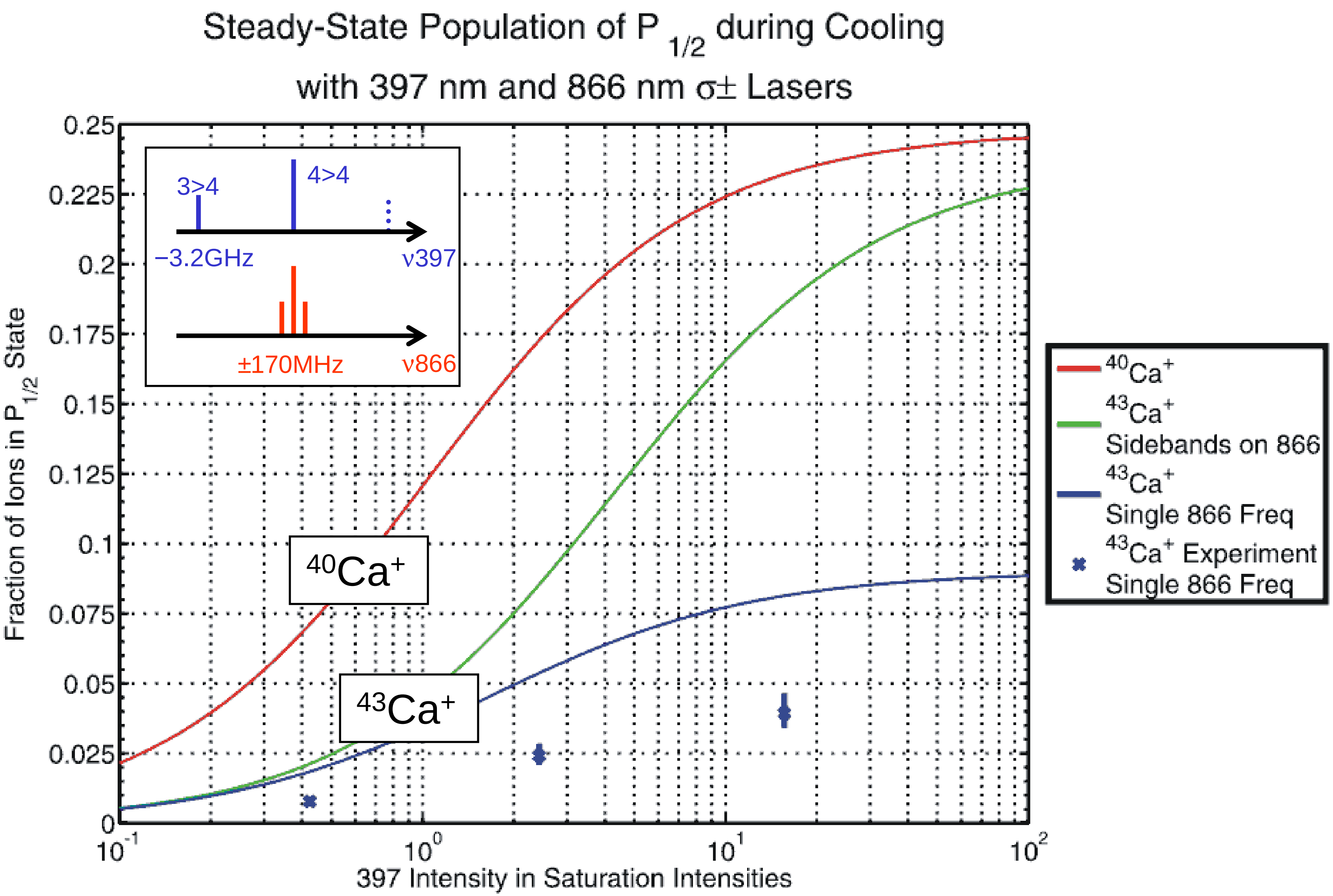


Presently get ~12.5 signal counts and ~0.1 background counts in 0.5ms, implies 0.9995 discrimination possible. P($D_{5/2}$ decay in 0.5ms)=1−0.9996.

Maximizing fluorescence in $^{43}Ca^+$



Maximizing fluorescence in $^{43}Ca^{+}$

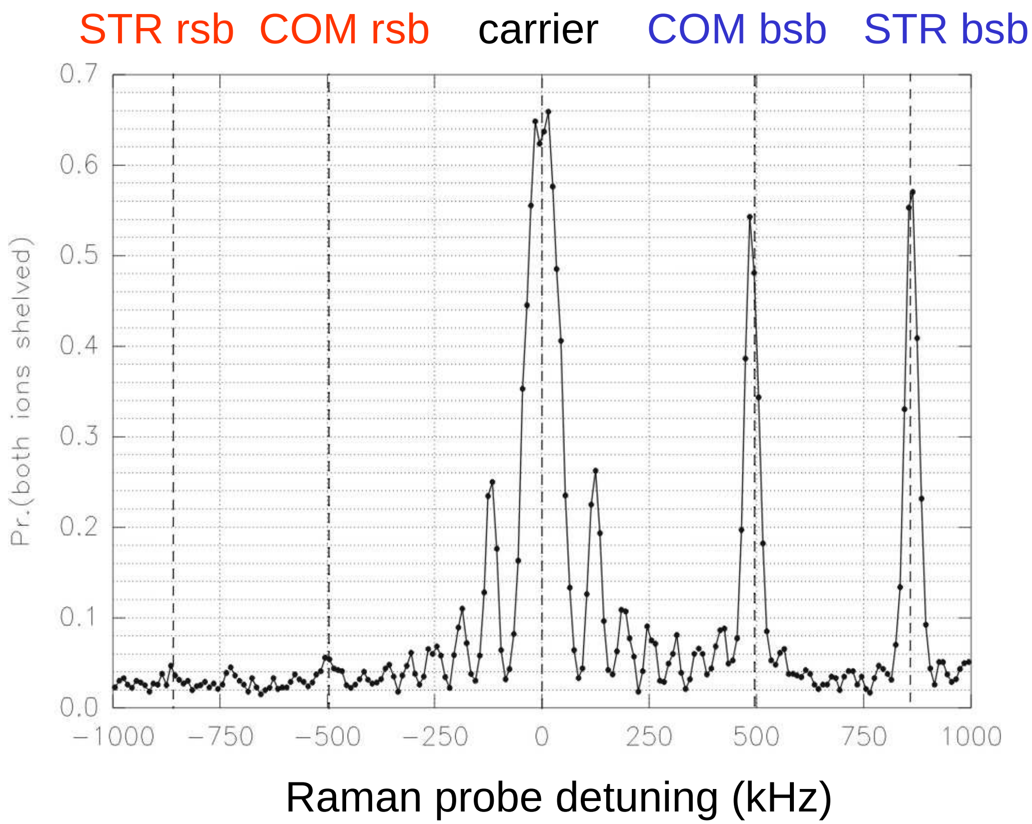


Cooling to ground motional state

Three stages:
- standard Doppler cooling to ~0.5mK → <n> ≈ 20
- continuous Raman sideband (Δ≈–150MHz) → <n> ≈ 0.5
- pulsed Raman sideband (Δ≈+30GHz) → <n> ≈ 0

Single ion:
<n>=0.02(2)
Heating rate:
3(1) phonons/sec
Motional coherence time of
|n=0> + |n=1> superposition:
200(50) ms

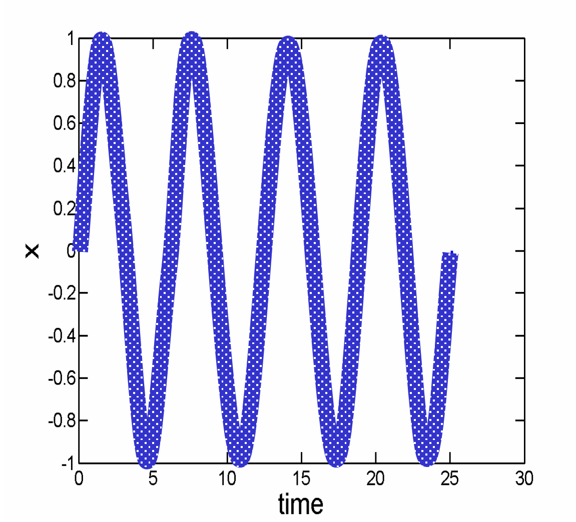
<u>Two-ion Raman spectrum</u> →
0.2ms Doppler
6ms continuous Raman
2x20 cycles pulsed Raman
$\langle n_{COM} \rangle = 0.2(1)$
$\langle n_{STR} \rangle = 0.08(5)$



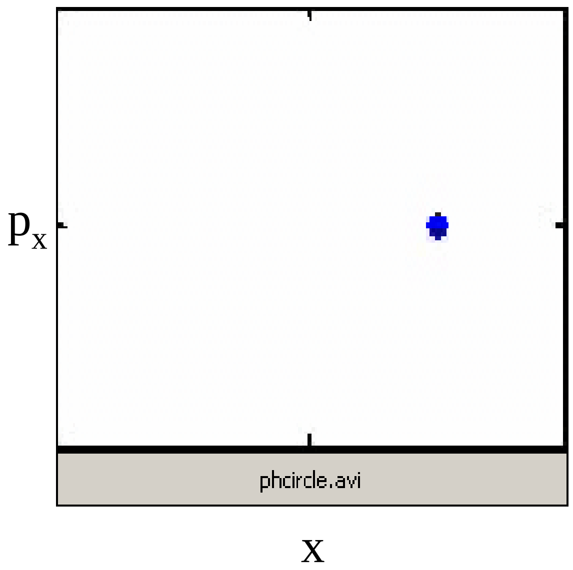
Cats

Quantum harmonic oscillator

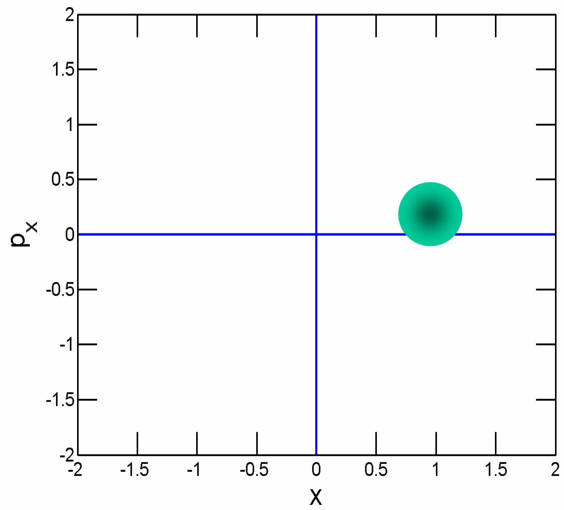
Lab space



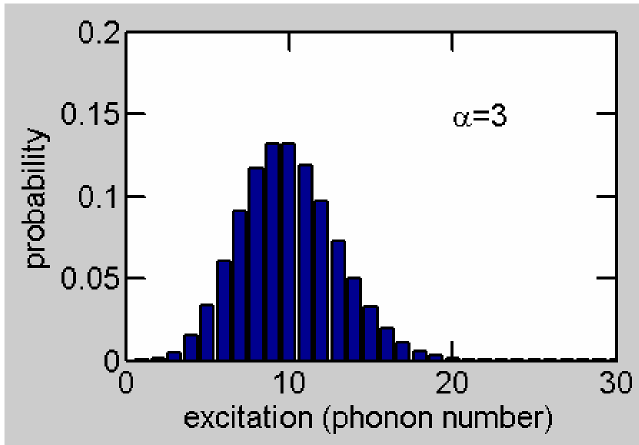
Phase space



Phase space, interaction picture



- If you displace the ground state in phase space, you get a *coherent state*.
- Each coherent state can be specified by a single complex number α.
- $|\alpha>$ can be expressed as a Poissonian distribution of number states with $<n> = |\alpha|^2$

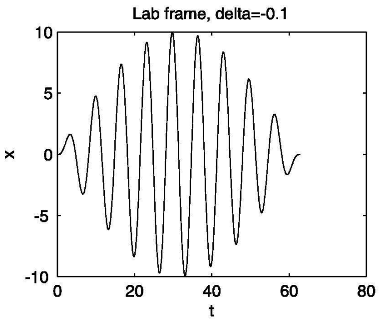


Forced quantum harmonic oscillator

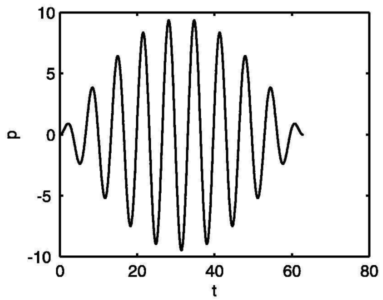
Apply a near-resonant oscillating driving force ("wobble"):
$F = F_0 \sin(\omega t)$ with $\omega = \omega_0 + \delta$

Lab space

x vs. t

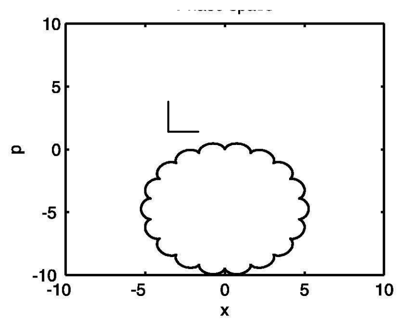


p vs. t



Phase space, rotating frame

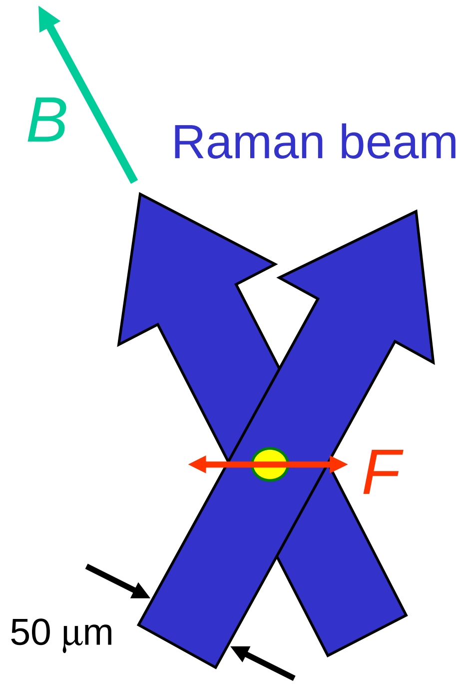
p vs. x

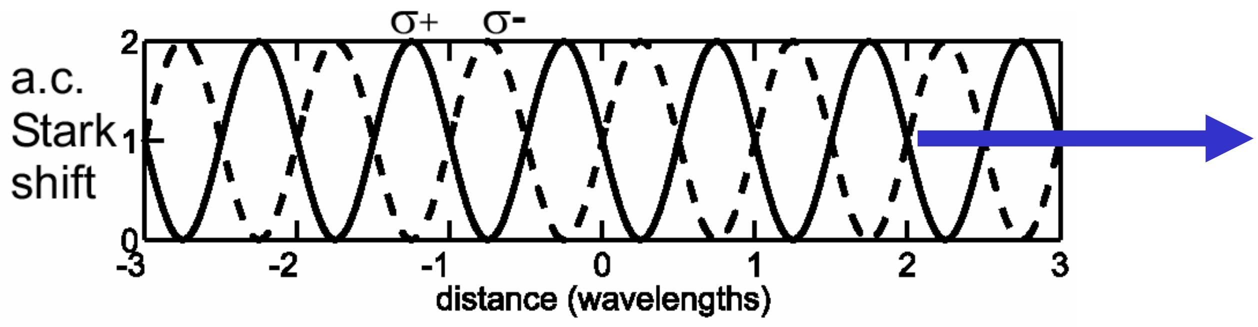


<u>Result</u>: displace the coherent state with $\alpha(t)$ following a circle

Time to complete the circle = $2\pi / \delta$
Maximum $|\alpha| \propto F_0 / \delta$

Spin-dependent "wobble" force

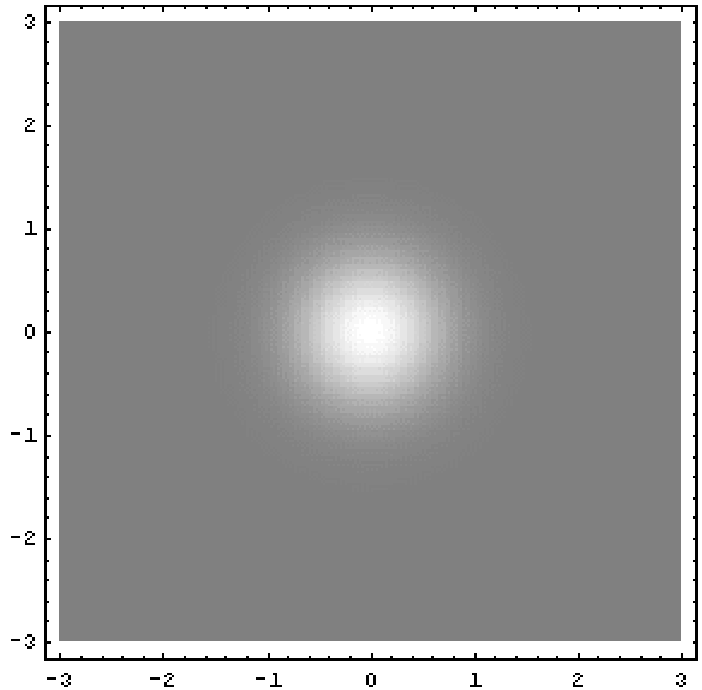




"Walking" standing wave

"Schrödinger Cat" experiment

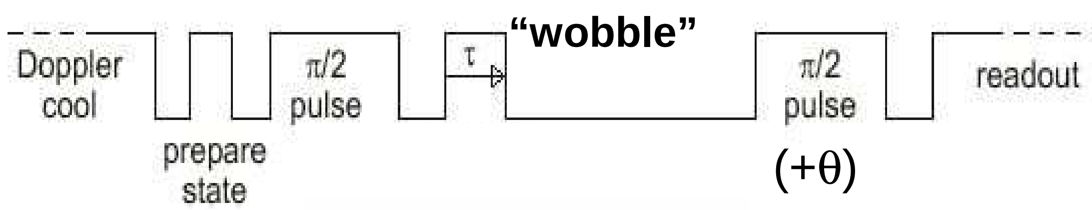
Create a superposition of 2 coherent states, then recombine them and show that their relative coherence is preserved:



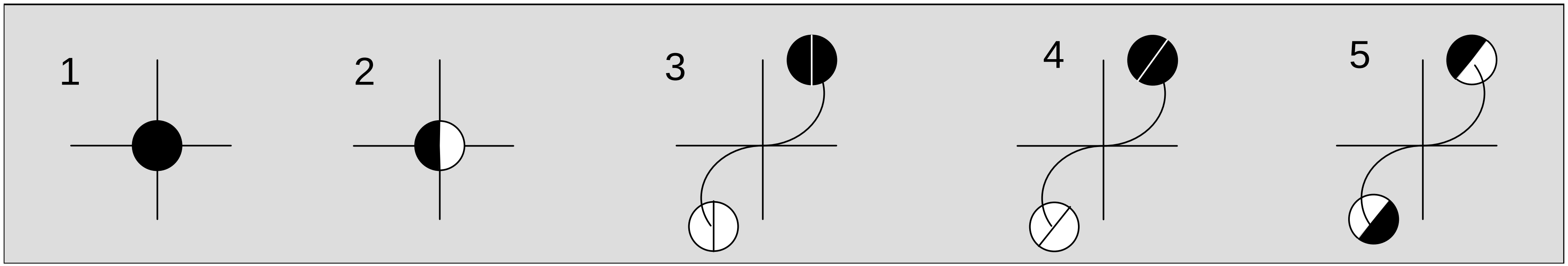
Example with small α

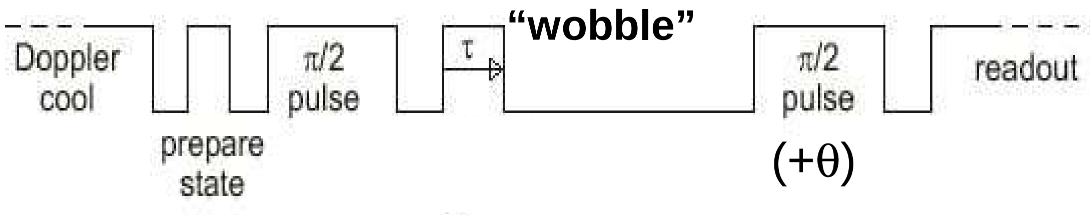
Similar to experiments with a trapped ion: C. Monroe *et al*, Science **272**, 1131 (1996)
and with atom+microwave cavity excitation: J.M.Raimond *et al*, Phys.Rev.Lett. **79**, 1964 (1997).

Sequence



1. cool to ground state and optically pump: $|\psi\rangle = |\downarrow\rangle\,|\alpha = 0\rangle$
2. $\pi/2$ pulse on spin state: $|\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|\downarrow\rangle + |\uparrow\rangle\right)|0\rangle$
3. apply spin-state-dependent force: $|\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|\downarrow\rangle\,|\alpha(t)\rangle + |\uparrow\rangle\,|{-\alpha(t)}\rangle\right)$
4. advance local oscillator by θ: $|\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|\downarrow\rangle\,|\alpha\rangle + e^{i\theta}\,|\uparrow\rangle\,|{-\alpha}\rangle\right)$
5. $\pi/2$ pulse on spin state: $|\psi\rangle = |\downarrow\rangle\left(\dfrac{|\alpha\rangle + e^{i\theta}\,|{-\alpha}\rangle}{2}\right) + |\uparrow\rangle\left(\dfrac{|\alpha\rangle - e^{i\theta}\,|{-\alpha}\rangle}{2}\right)$
6. measure spin: $\mathrm{P}(\uparrow) = \dfrac{1}{2}\left(1 + \mathrm{Re}(e^{i\theta}\,\langle\alpha|{-\alpha}\rangle\right) = \dfrac{1}{2}\left(1 + \cos(\theta)\; e^{-2|\alpha|^2}\right)$



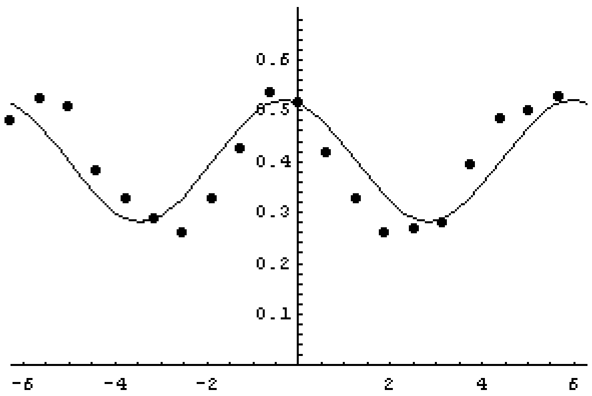


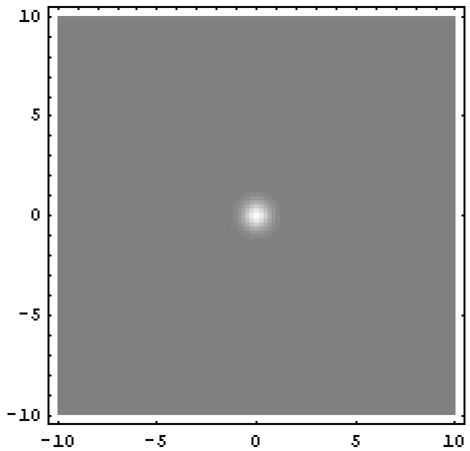
Data

Interpretation

no wobble force

Prob (spin ↑)





-2π

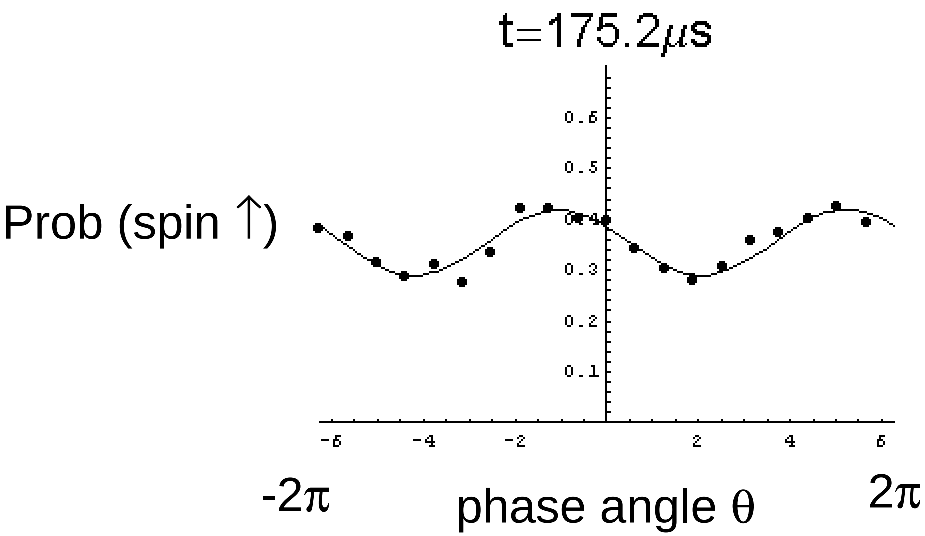
phase angle θ

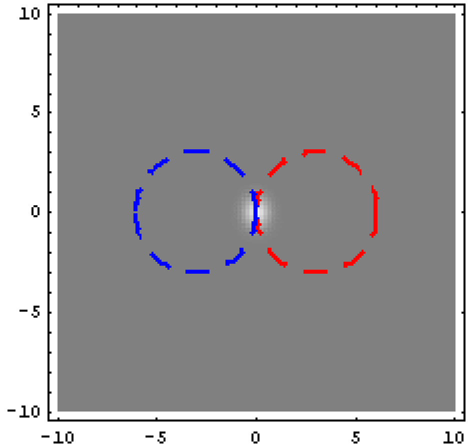
2π

Each point = 1000 repeats of the experiment

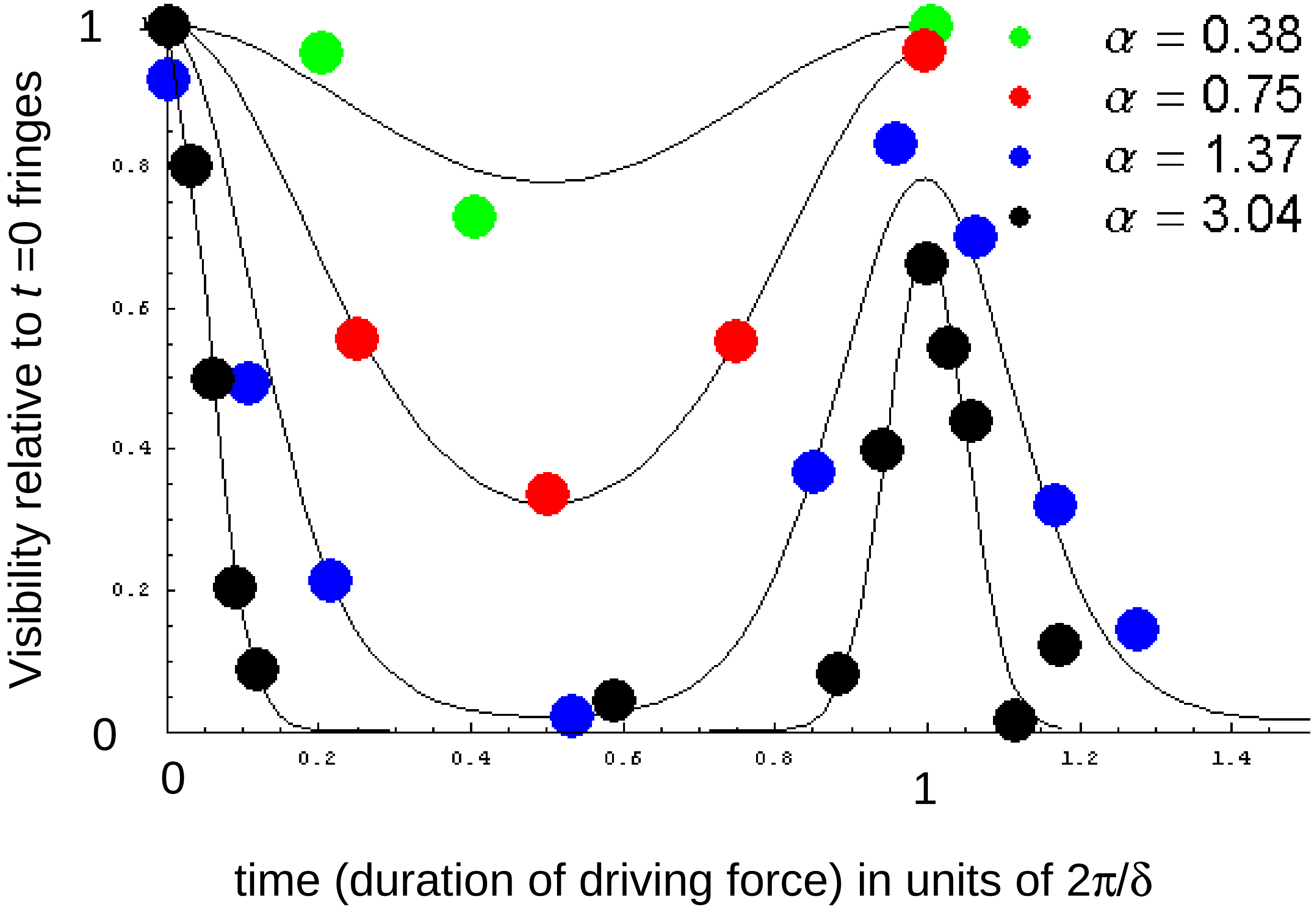
Data

Interpretation





Cats of all (smallish) sizes...

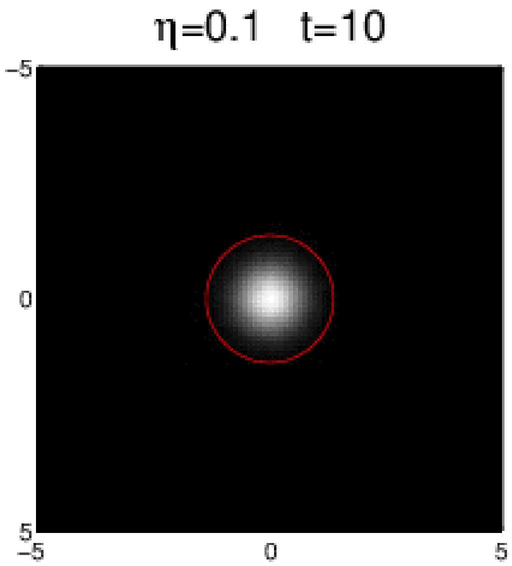


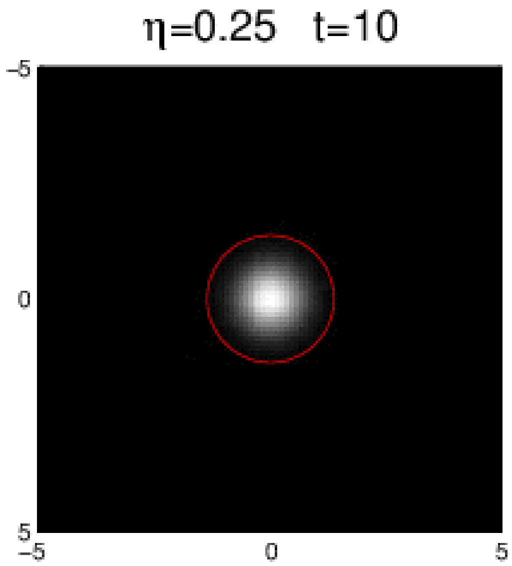
Cats beyond the Lamb-Dicke regime

For large spatial excursions the driving force is really:

$F = F_0 \sin(kx - \omega t)$

Outside Lamb-Dicke regime (large η) cannot neglect kx term.

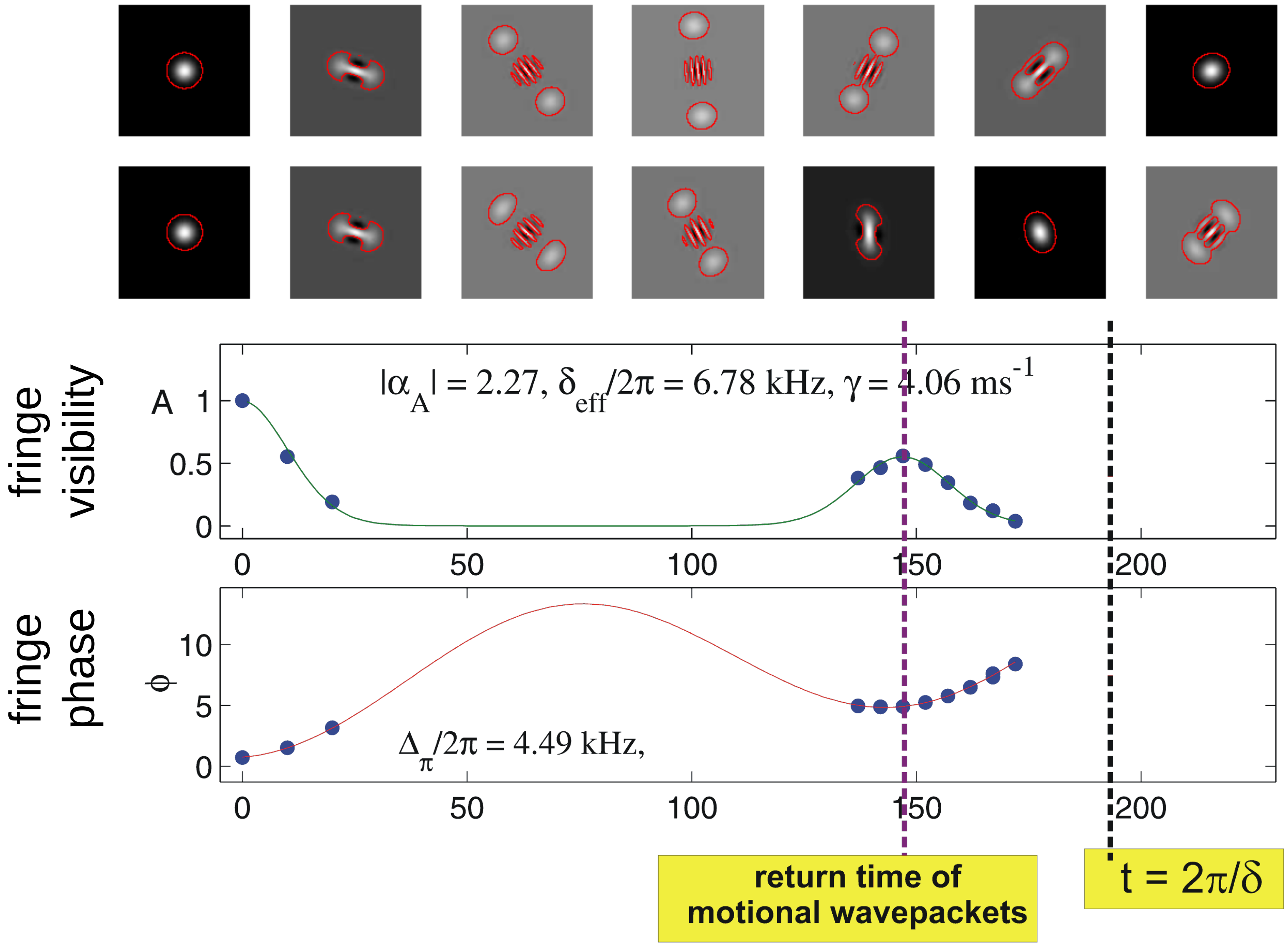




Force amplitude scaled in both cases to give same “expected” α_{MAX}

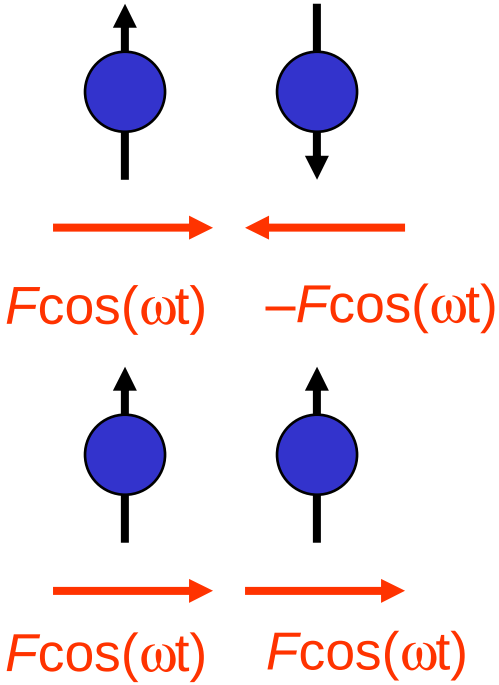
For η=0.25, see squeezing and “barrier” effect

Cats beyond the Lamb-Dicke regime



Gates

“Wobble” gate for 2-ion entanglement



Forces in opposite directions: forces STRETCH motion (ω close to resonance): coherent drive gives phase shift

Forces in same direction: tries to force COM motion, but ω far-detuned: no effect

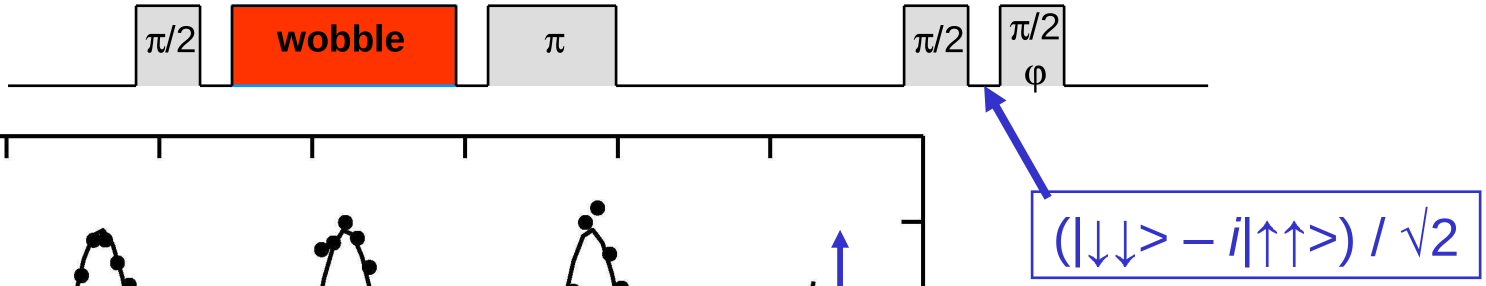
Truth table:

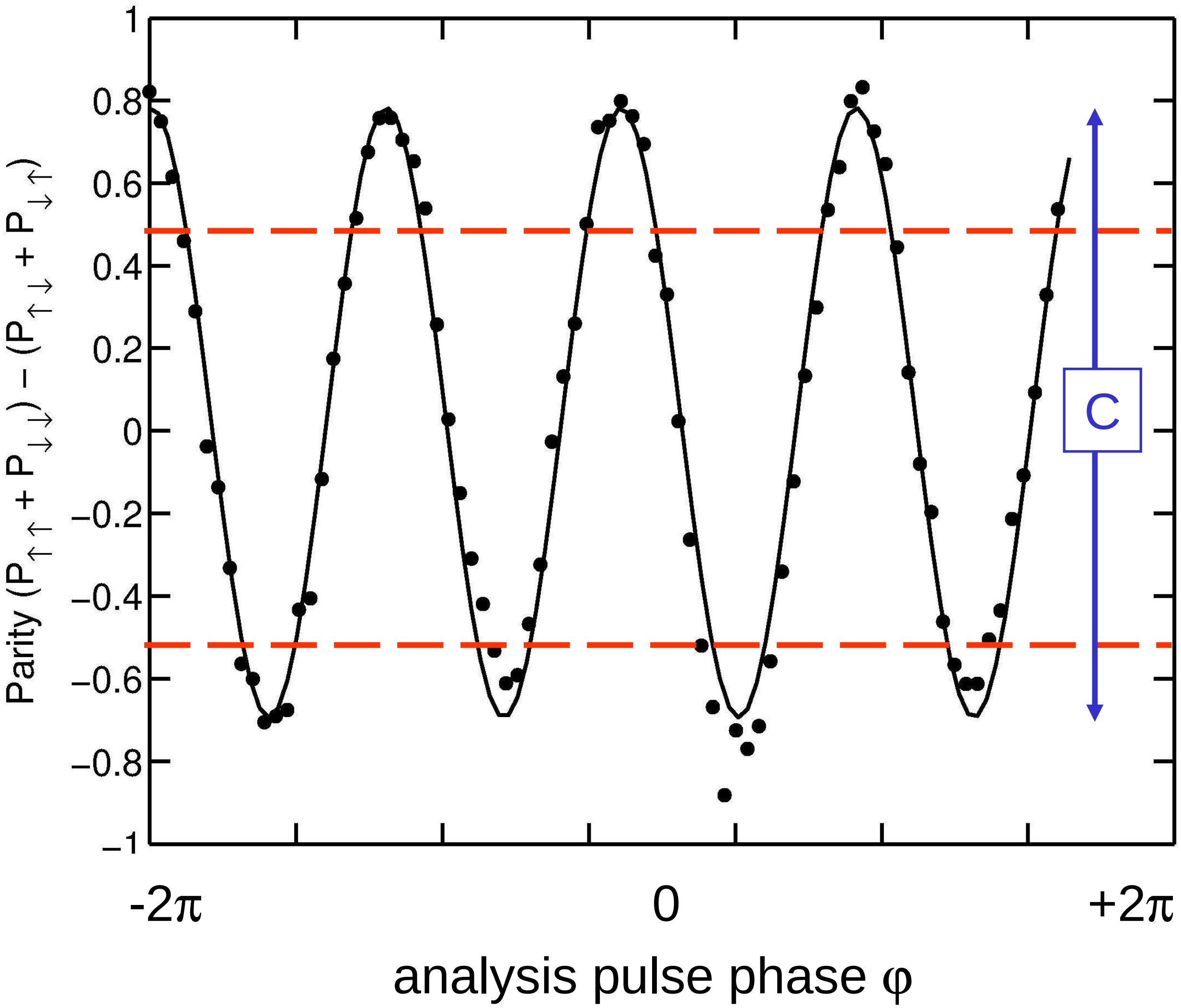
$$
\begin{aligned}
|\downarrow\downarrow\rangle &\rightarrow |\downarrow\downarrow\rangle \\
|\uparrow\downarrow\rangle &\rightarrow e^{i\pi/2}|\uparrow\downarrow\rangle \\
|\downarrow\uparrow\rangle &\rightarrow e^{i\pi/2}\,|\downarrow\uparrow\rangle \\
|\uparrow\uparrow\rangle &\rightarrow |\uparrow\uparrow\rangle \\
&= e^{i\pi}(e^{i\pi/2}|\uparrow\rangle)(e^{i\pi/2}|\uparrow\rangle)
\end{aligned}
$$

“Controlled-phase” gate: equivalent to CNOT up to single-qubit phases

Leibfried *et al.*, Nature 2003: F=97% !

"Wobble" gate result





$^{40}Ca^+$ spin qubit

stretch mode 866kHz
ion spacing 9μm (=22λ)

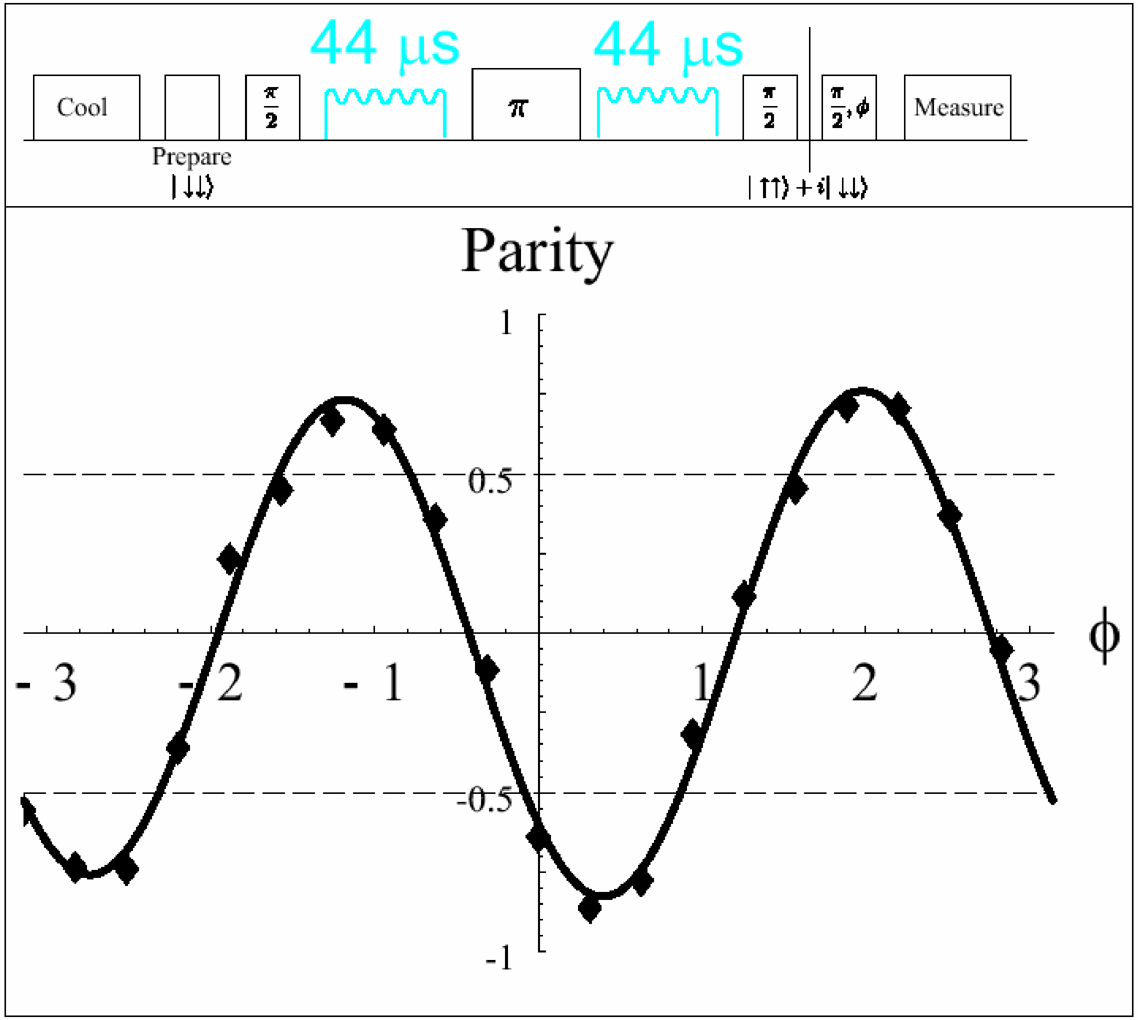
$\langle n_{COM} \rangle \approx 0.35$
$\langle n_{STR} \rangle \approx 0.1$

gate time $\tau = 77\mu s$
$= 67 / \nu_{STR}$

fidelity = 75(5)%
from $(P_{\uparrow\uparrow} + P_{\downarrow\downarrow} + C)/2$

Feb 2005

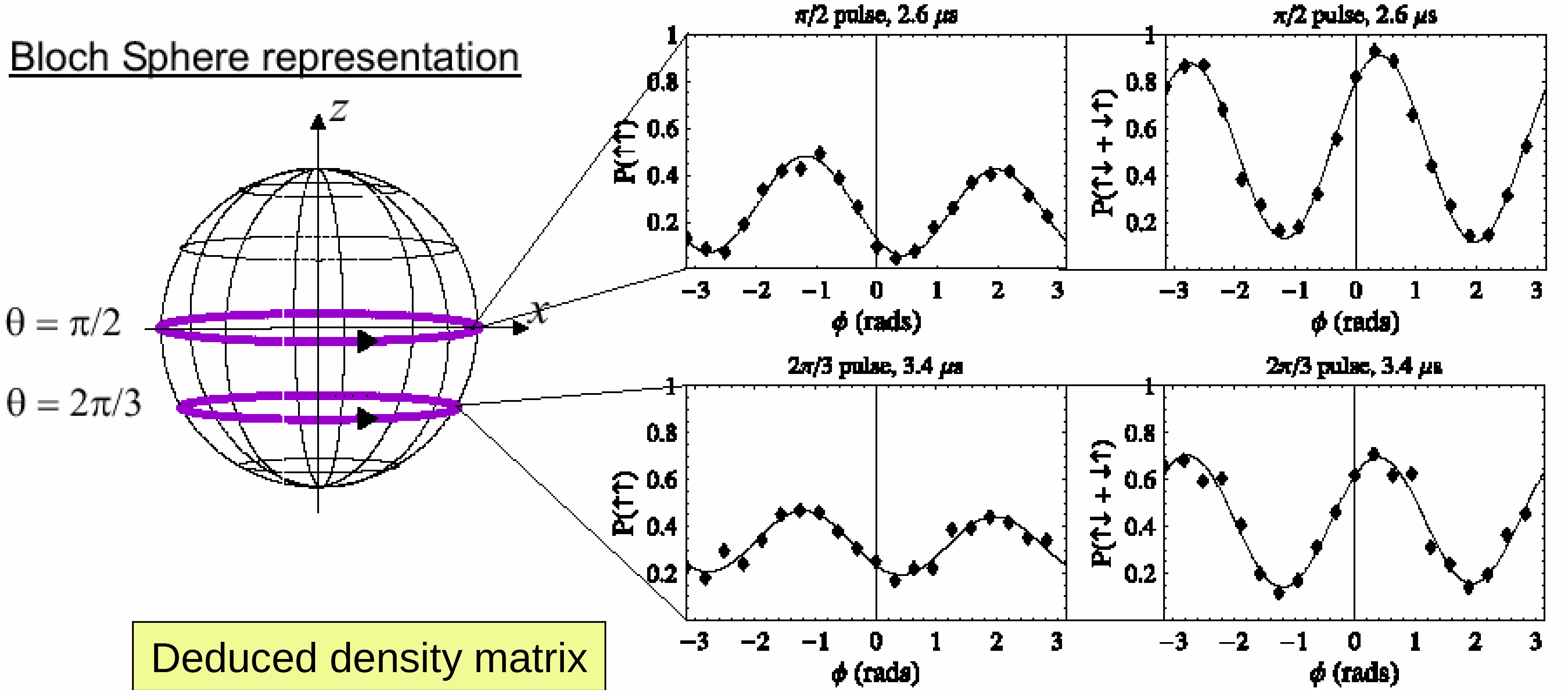
"Double Wobble" result



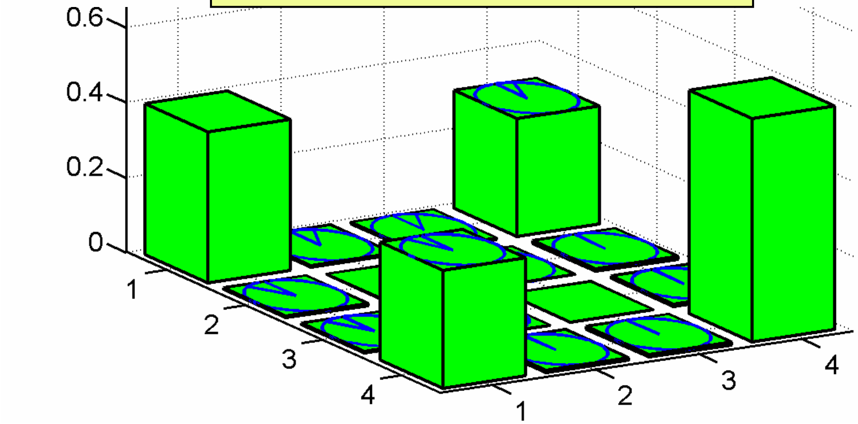
- use twin loop to eliminate single-qubit phase shifts
- fidelity now 82(2)%
- ~12% infidelity due to photon scattering (Δ = 30 GHz)
- also imbalanced light intensity, motional decoherence…

June 2005

Tomography of entangled state



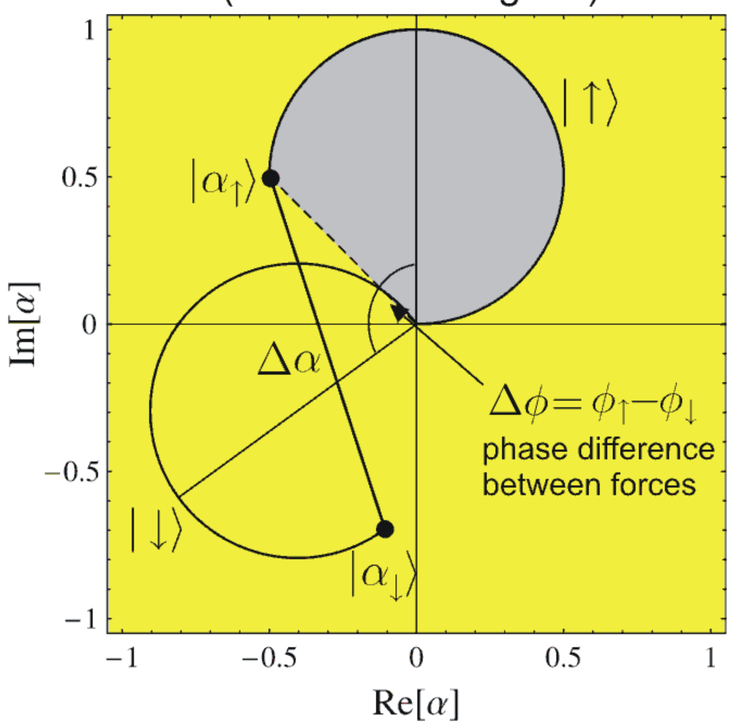
Deduced density matrix



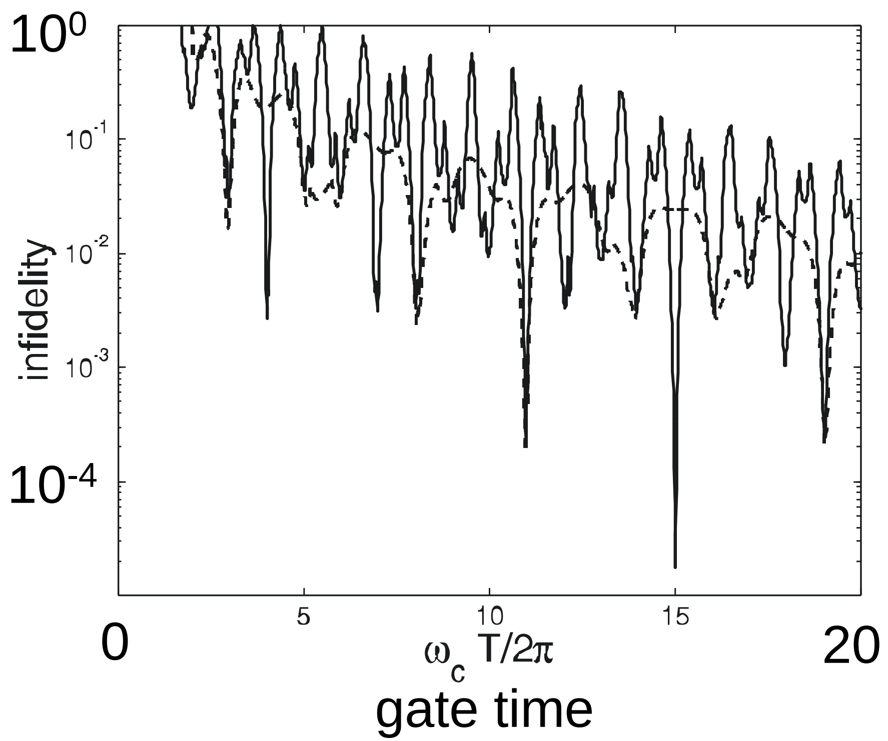
hence entanglement of formation
$E = 0.52$

Fast gates with the "wobble" force?

Rotating frame phase space picture
(Lamb-Dicke regime)



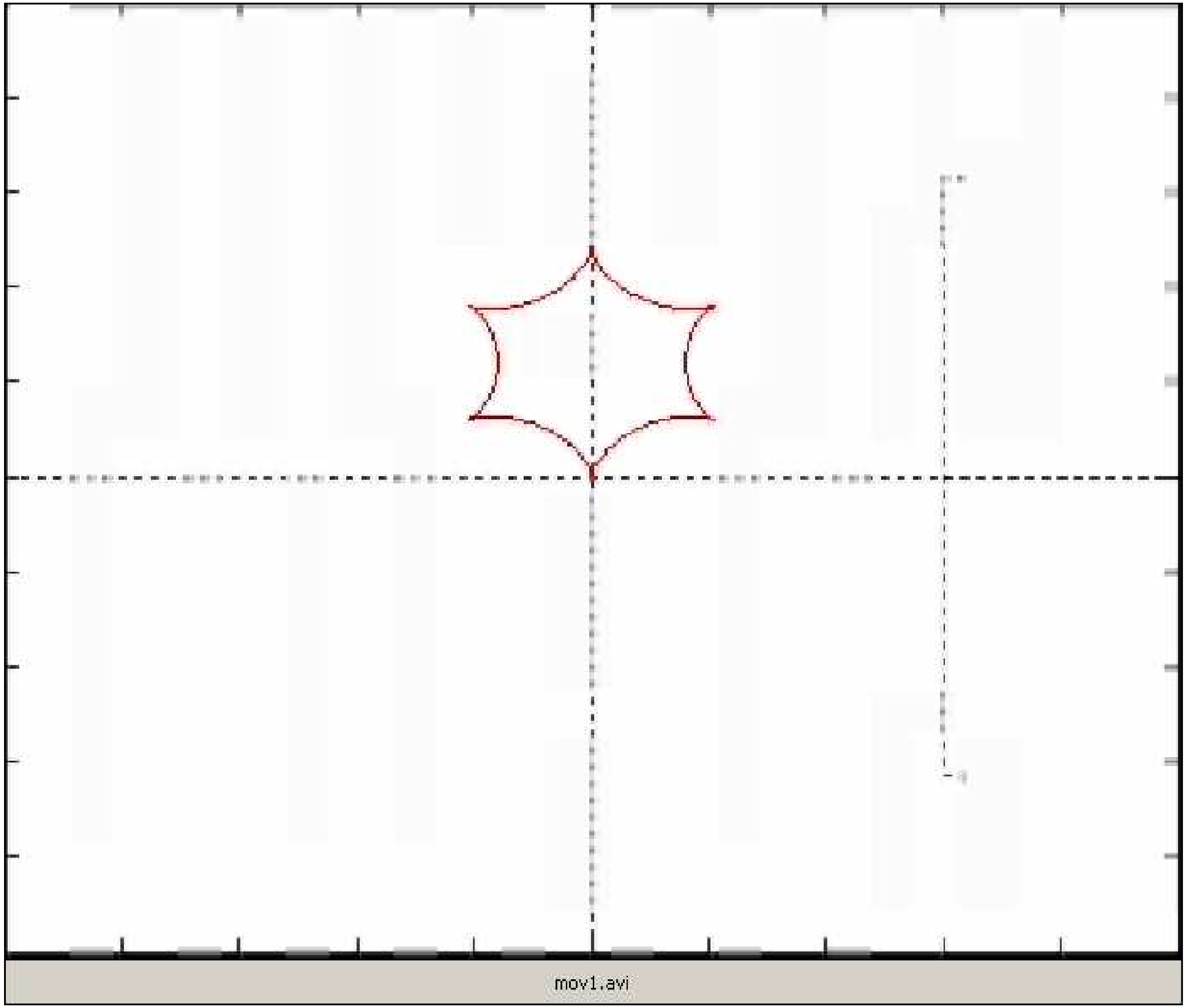
Rotating wave approximation
breaks down at large detunings



η_c=0.1, <n_c>=<n_s>=1

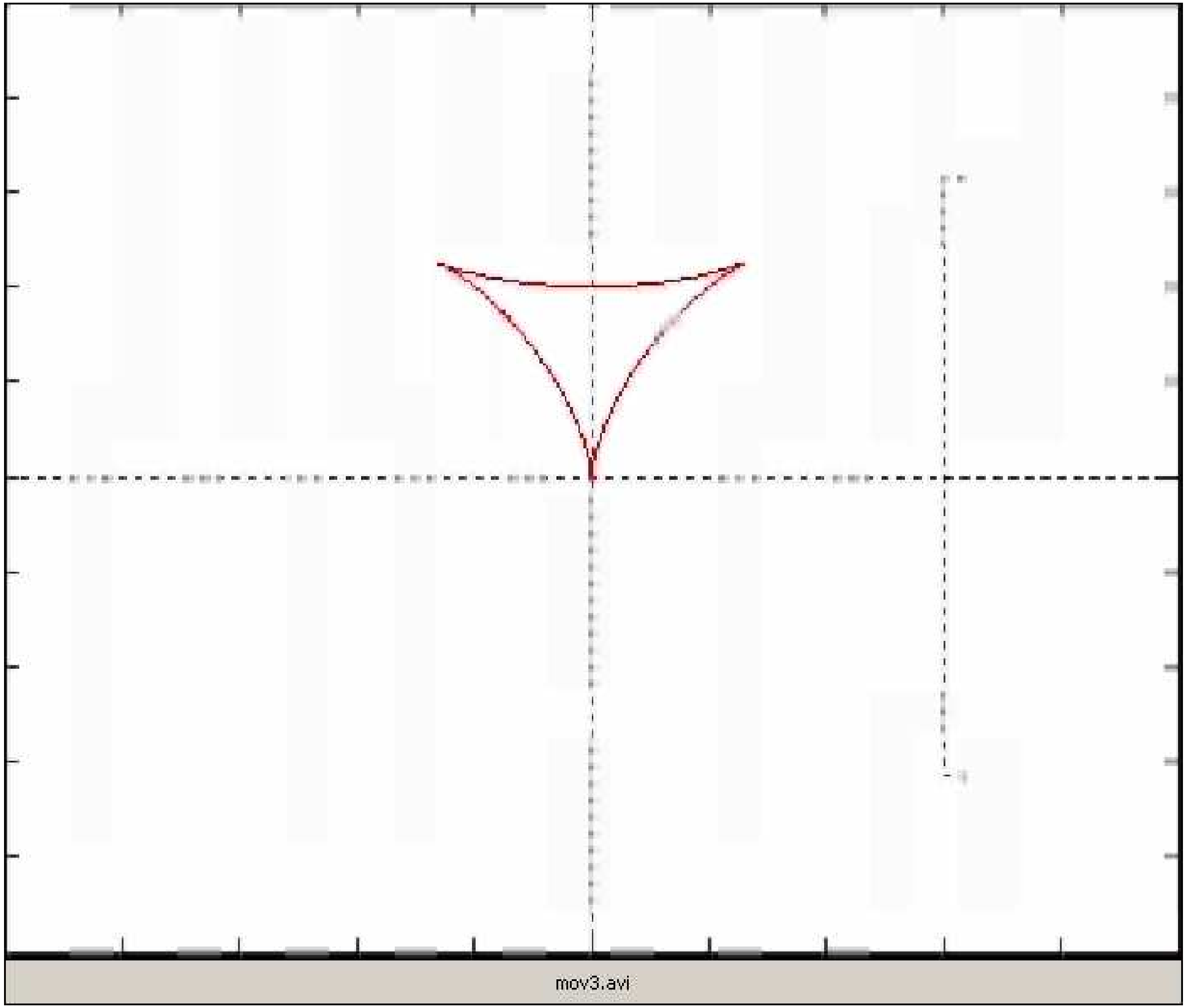
Phase gate is insensitive to
fluctuations in walking-wave phase

Fast gates with the "wobble" force?



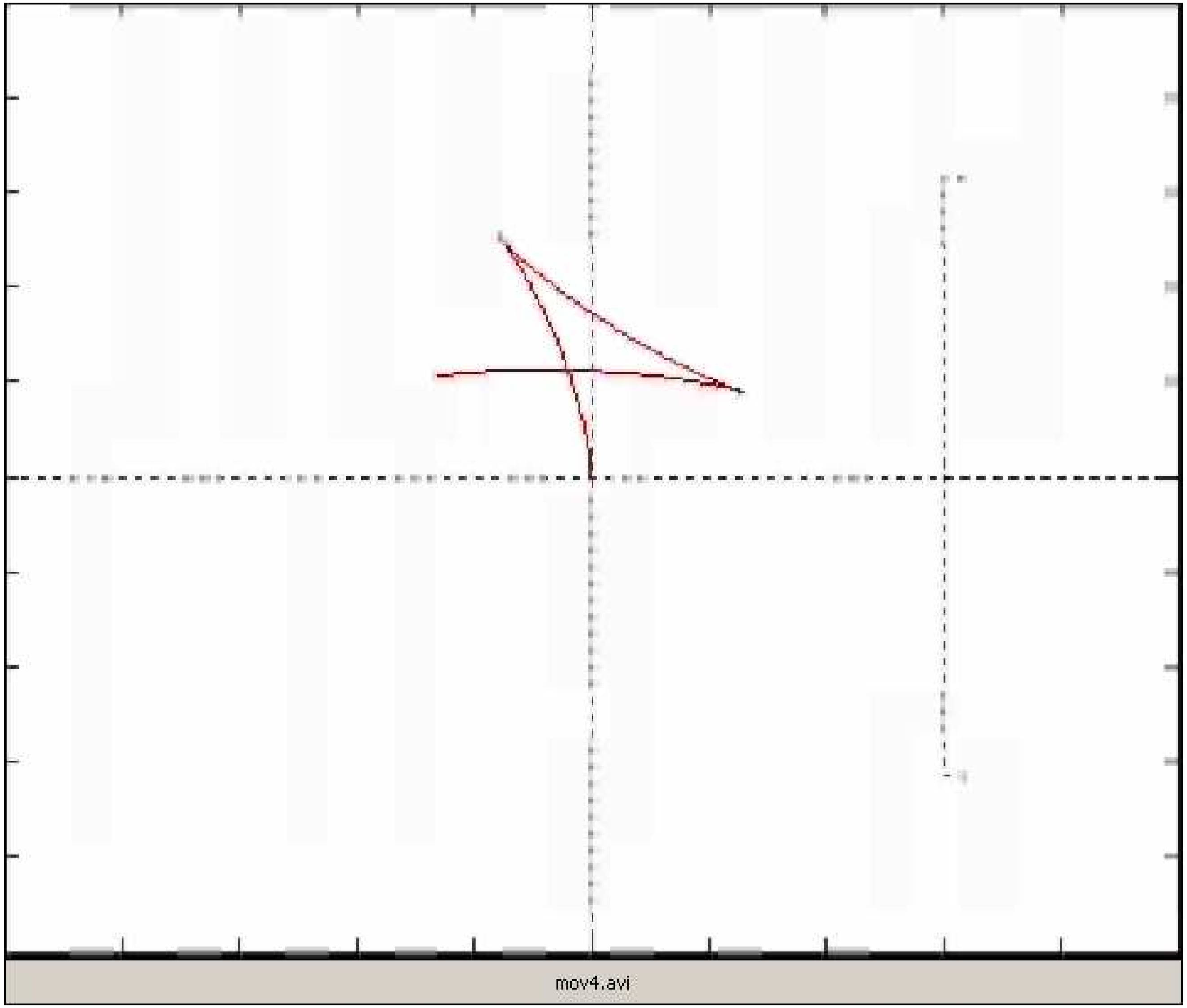
STRETCH
$\omega=1.5\omega_s$
single pulse

Fast gates with the "wobble" force?



STRETCH
$\omega = 3\omega_s$
single pulse

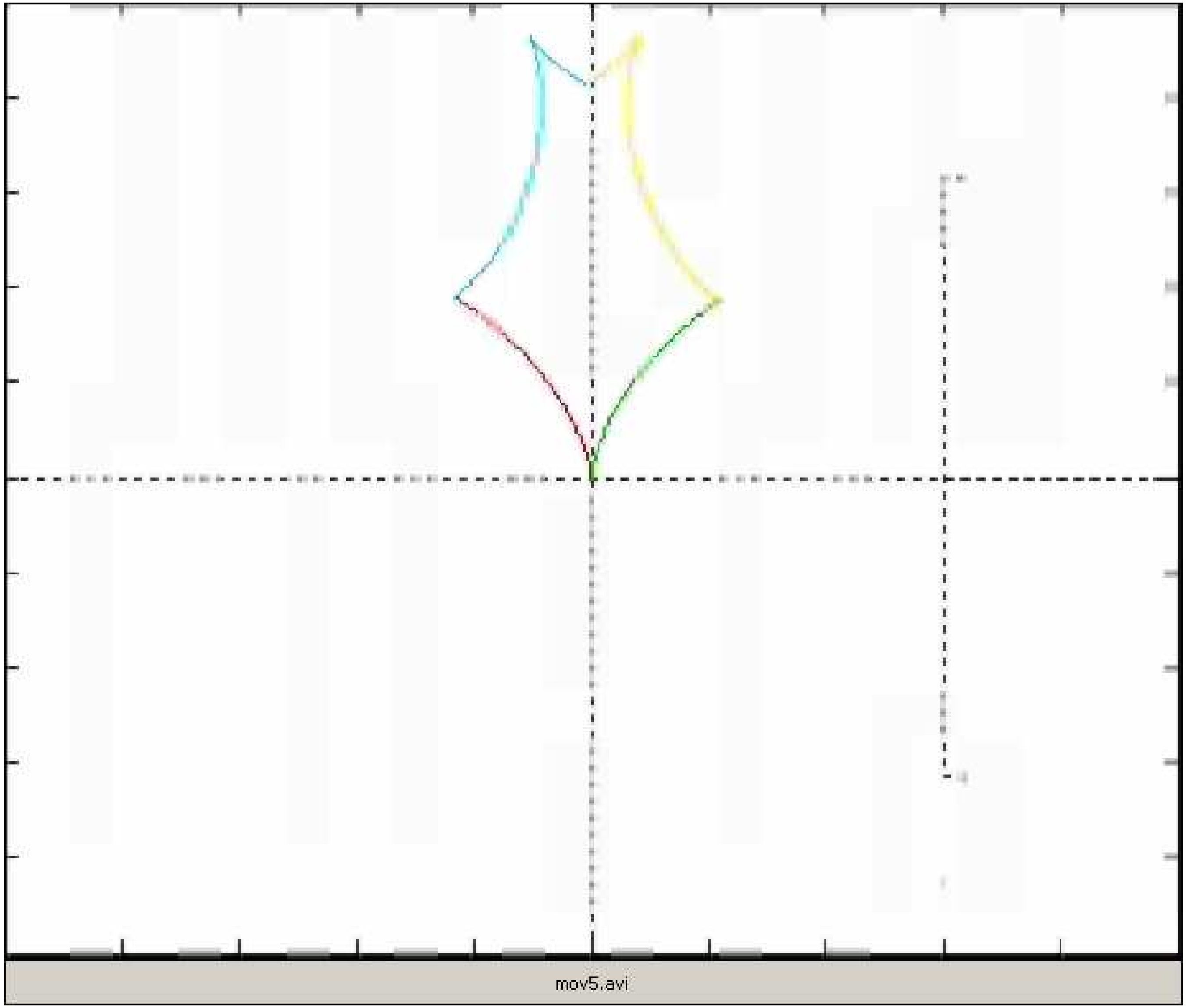
Fast gates with the "wobble" force?



COM
$\omega = 3\omega_s$
single pulse

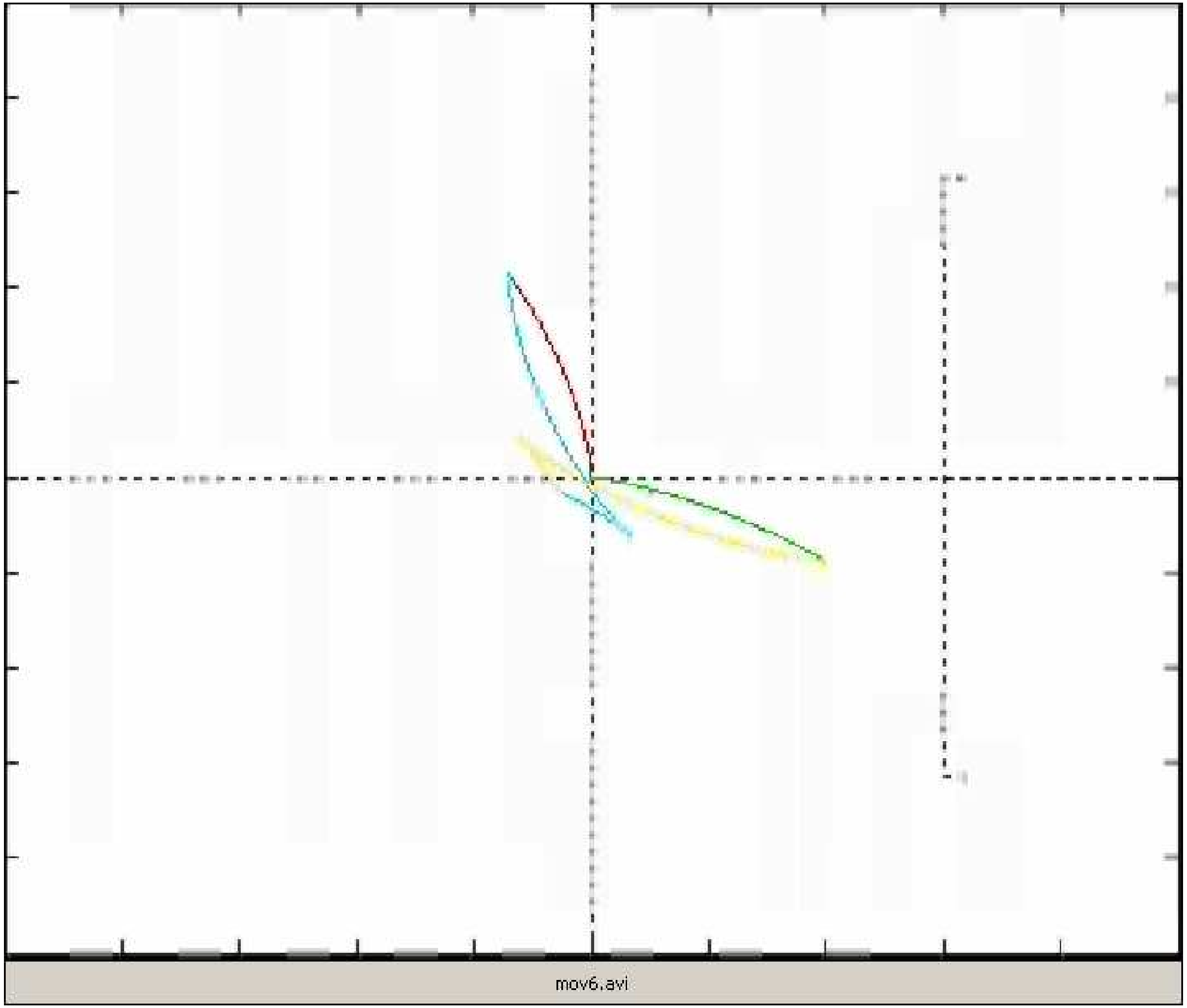
orbit not closed!

Fast gates using pulsed force



STRETCH
$\omega=4.04\omega_s$
4 pulses

Fast gates using pulsed force



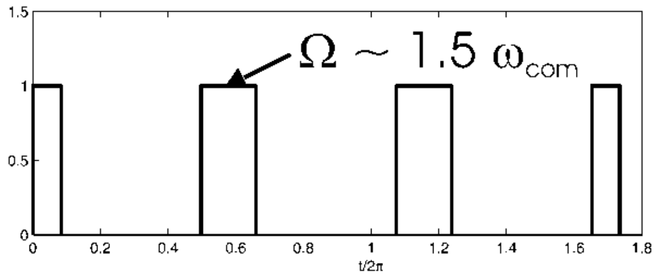
COM
$\omega=4.04\omega_s$
4 pulses

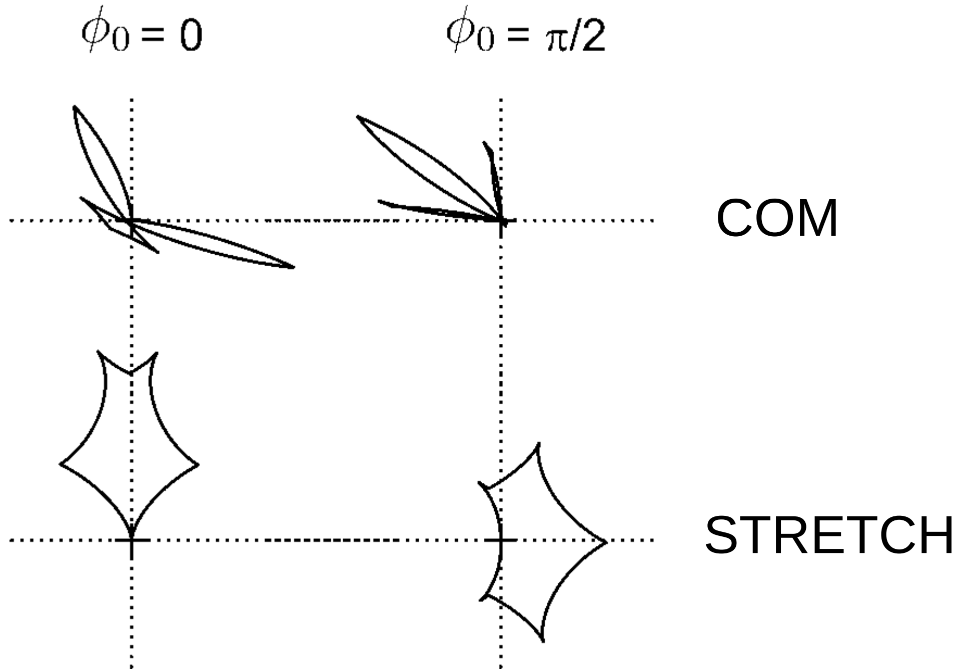
Fast gates using pulsed force

4 pulses (simple)

all $\omega \sim 4\ \omega_{com}$

$t(\text{gate}) = 1.75\ 2\pi/\omega_{com}$





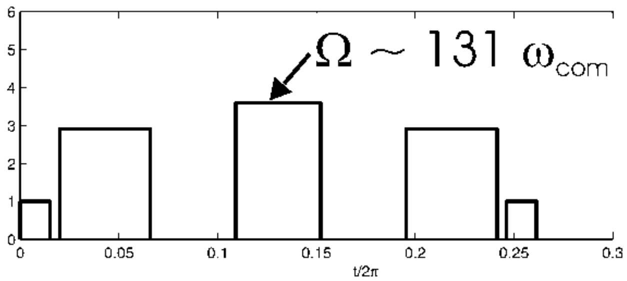
fidelity = 0.99999
(excluding photon scattering, heating…)

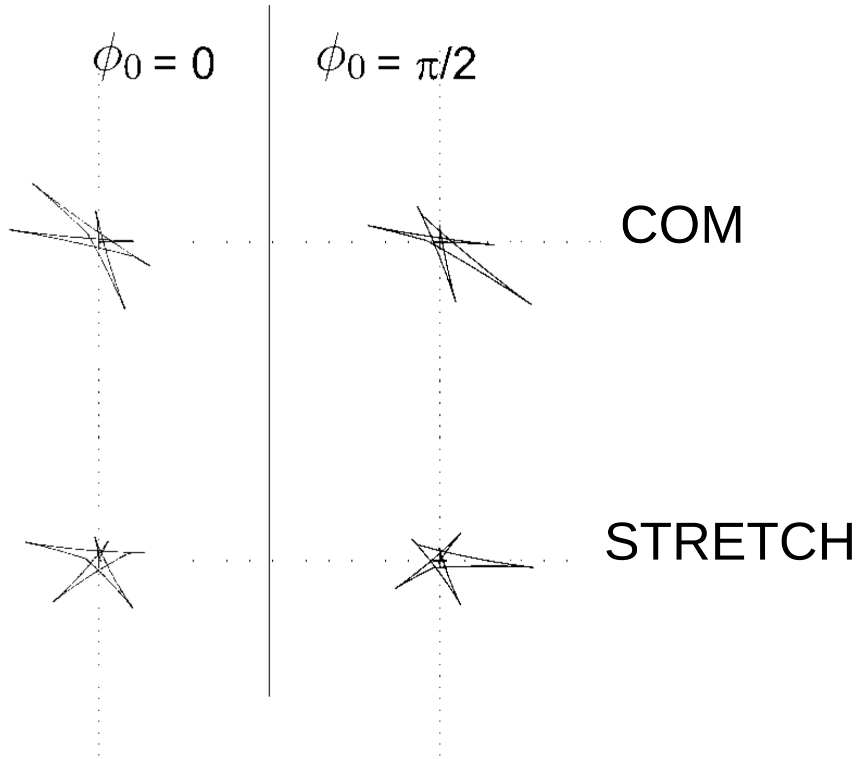
Fast gates using pulsed force

5 pulses

freq. $\omega \sim 20\ \omega_{com}$

$t(\text{gate}) = 0.27\ 2\pi/\omega_{com}$

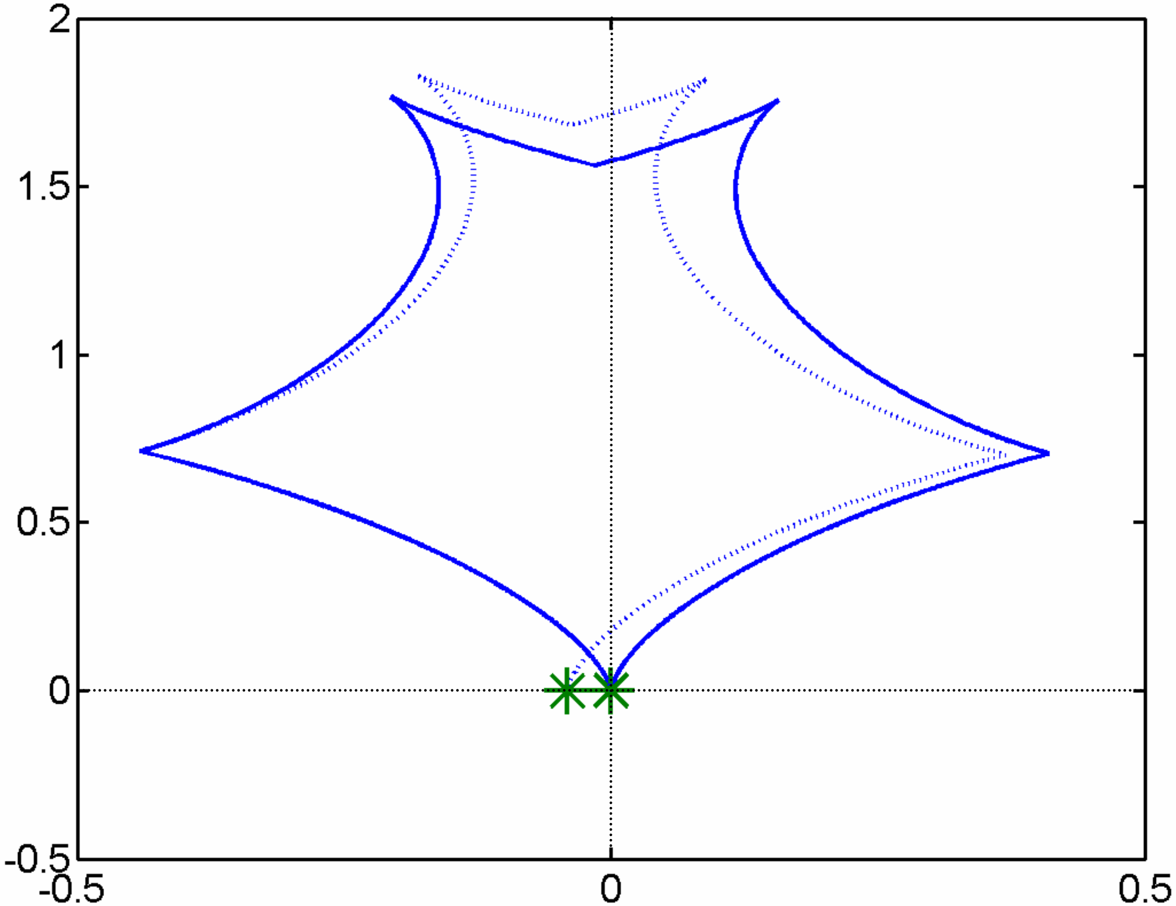




fidelity = 0.999 999 99

(excluding photon scattering, heating…)

Fast gates using pulsed force



.... Lamb-Dicke trajectory

4 pulse sequence with η_c=0.1
fidelity drops from 0.99999 to 0.9988 (so far…)

Fast gates using pulsed force

$$\int \Omega(t)dt \text{ vs. gate time}$$